The Perfect System:

Alpha Centauri A & B as Ideal Calibrators of Energy Transport Formalisms in Stellar Evolution Models

The Alpha Centauri System: Towards new worlds
26 June 2023

Dr Meridith Joyce

Marie Curie Widening Fellow: MATISSE
CSFK Konkoly Observatory, Budapest

MESA Developers

Incoming Assistant Professor
University of Wyoming

@MeridithJoyceGR

www.meridithjoyce.com

github.com/mjoyceGR

meridith.joyce@csfk.org

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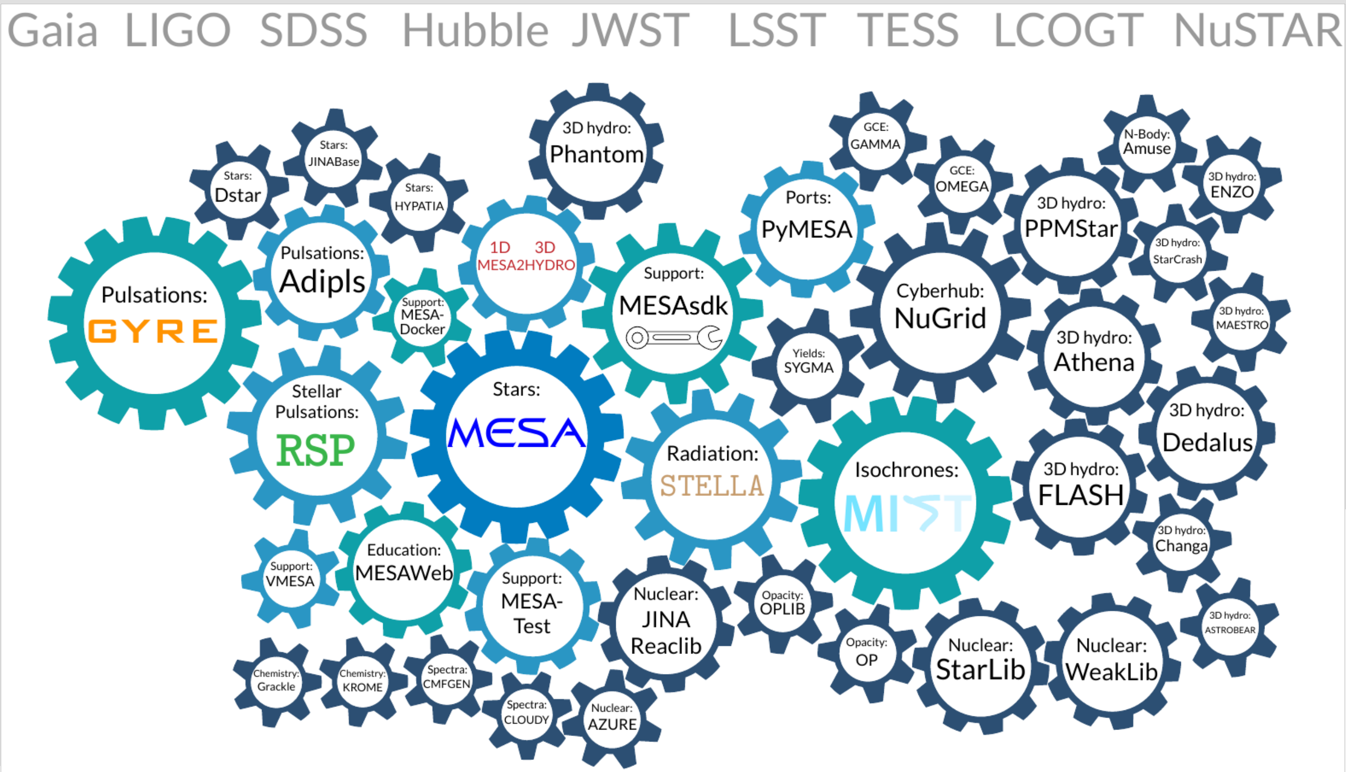
→ **the most important models in astronomy are stellar tracks & isochrones**

A ***stellar structure and evolution program*** solves coupled differential equations in **radius** and **time** to provide a model of a star's life



GYRE

D S E P

Dartmouth Stellar Evolution Program

Four kinds of stellar models lay the foundation:

(1) Stellar tracks

(2) Stellar profiles

(3) Stellar oscillations

(4) Isochrones

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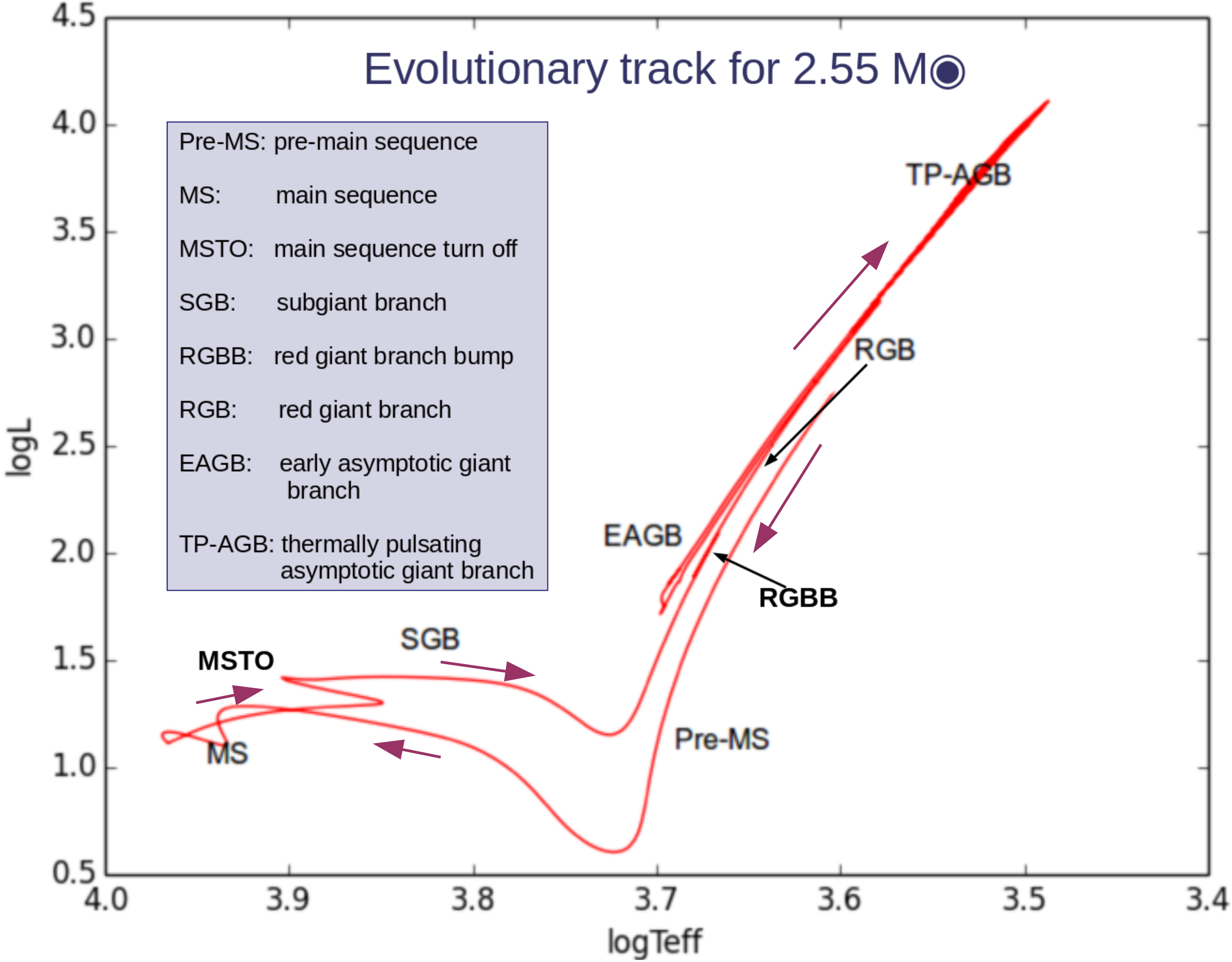
Output: Stellar track



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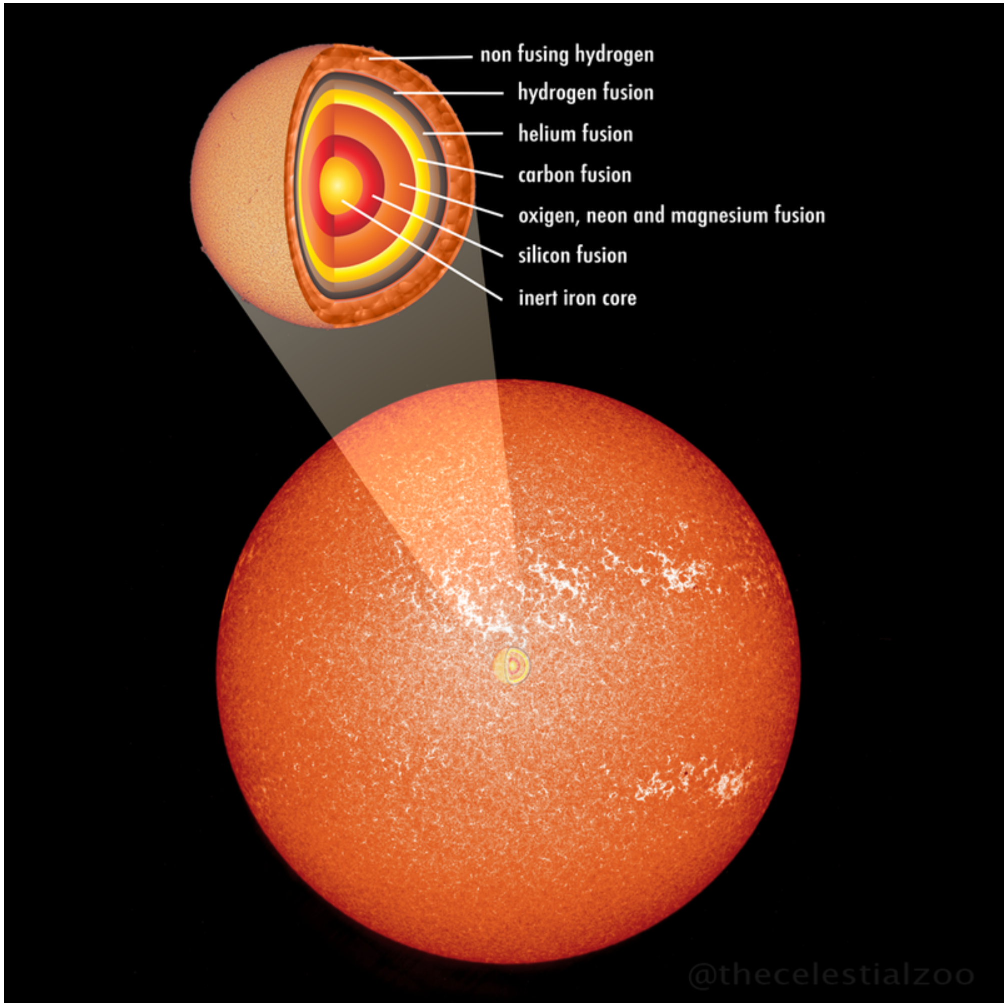
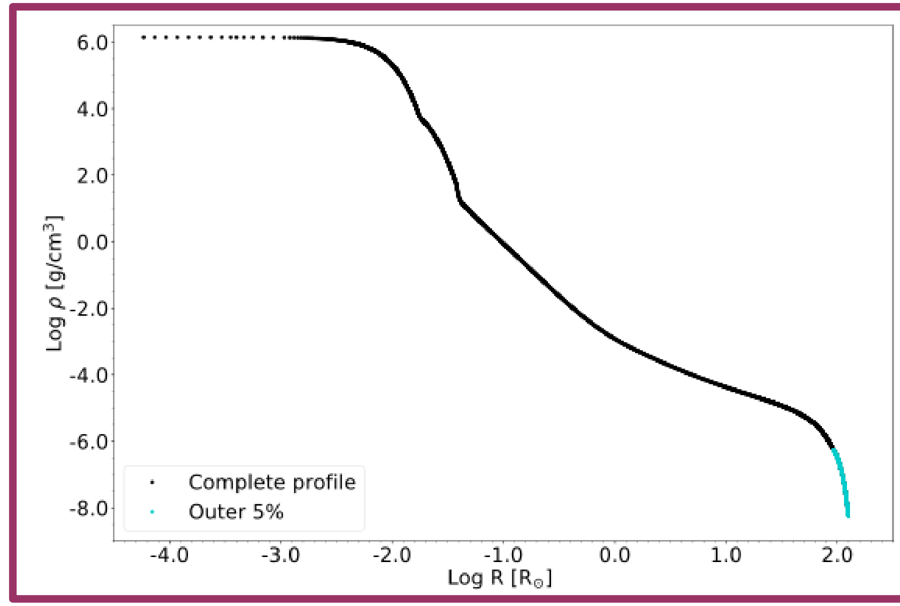
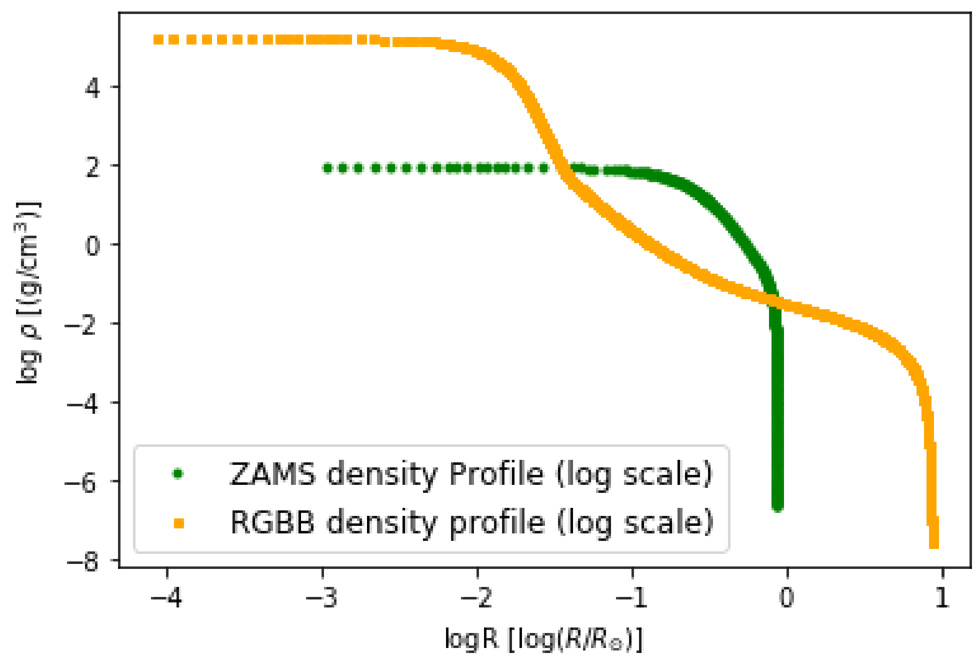
Stellar profile (density structure)







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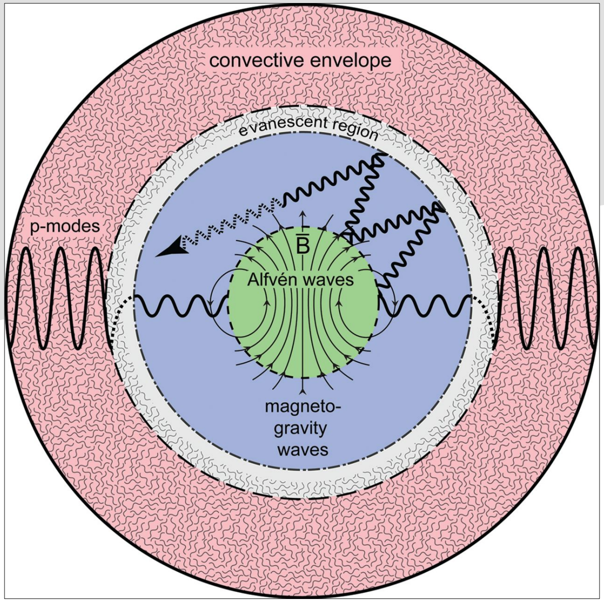
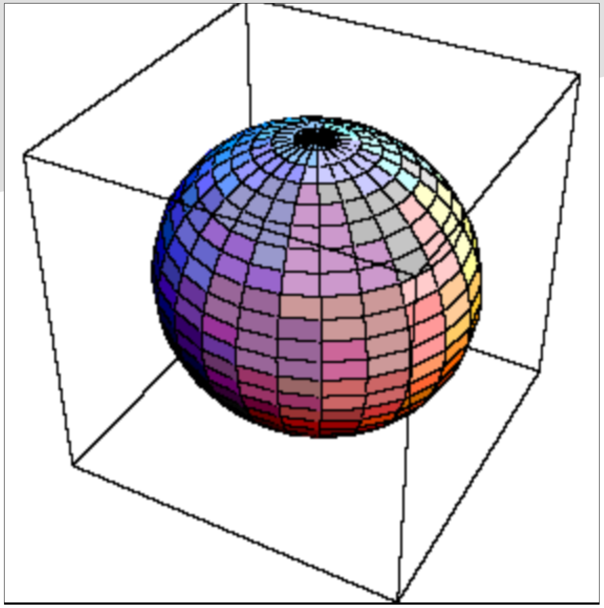
Key Concept: Stars pulsate!

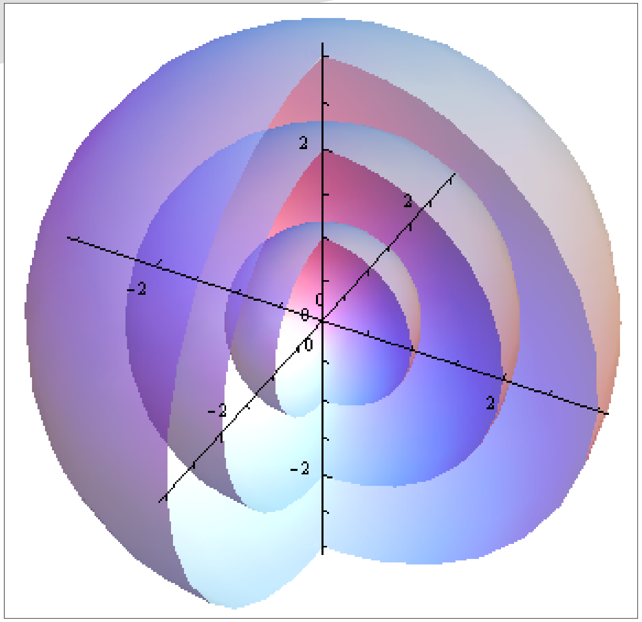
They “ring” like bells, in response to physical mechanisms causing waves inside them

The frequencies (pitches) at which they ring can tell us what they’re made out of

The use of these pulsations to learn about stellar structure is called **asteroseismology**







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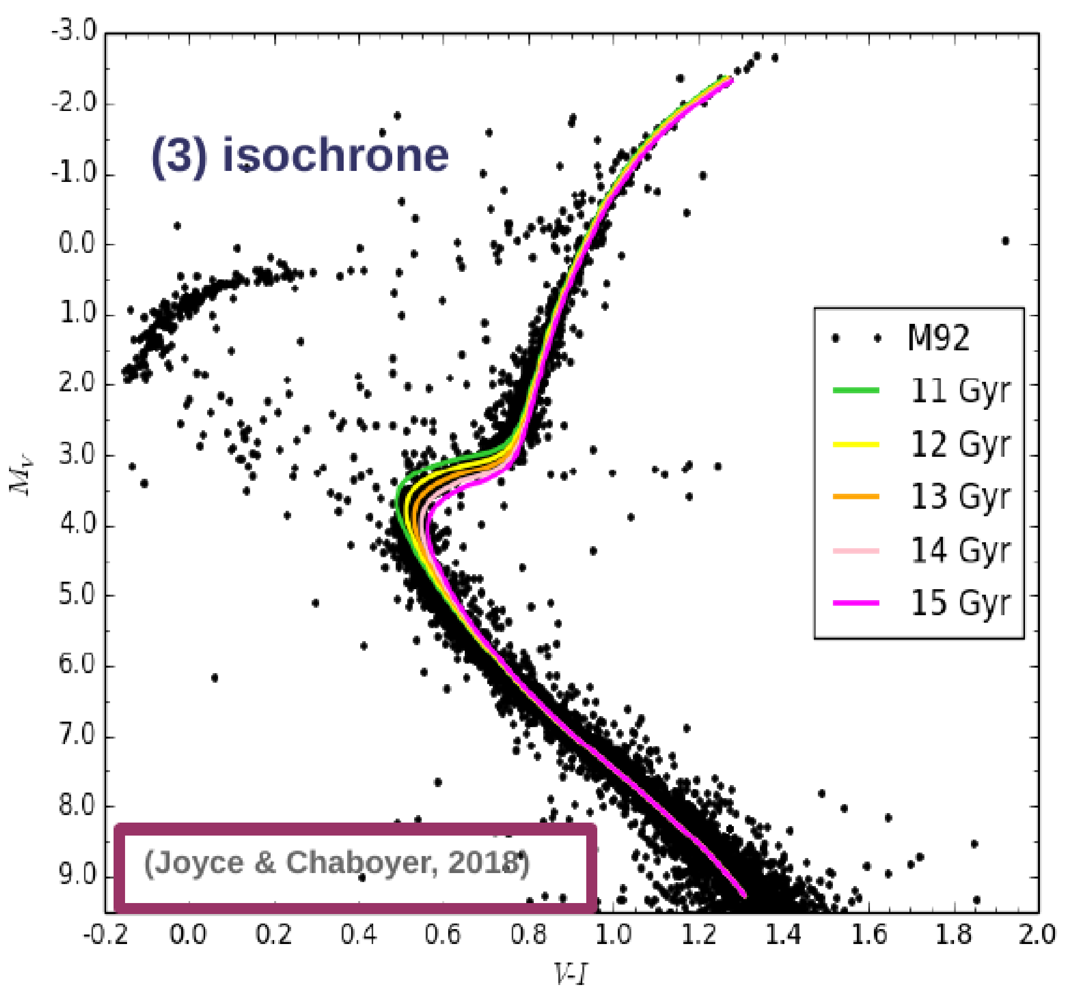
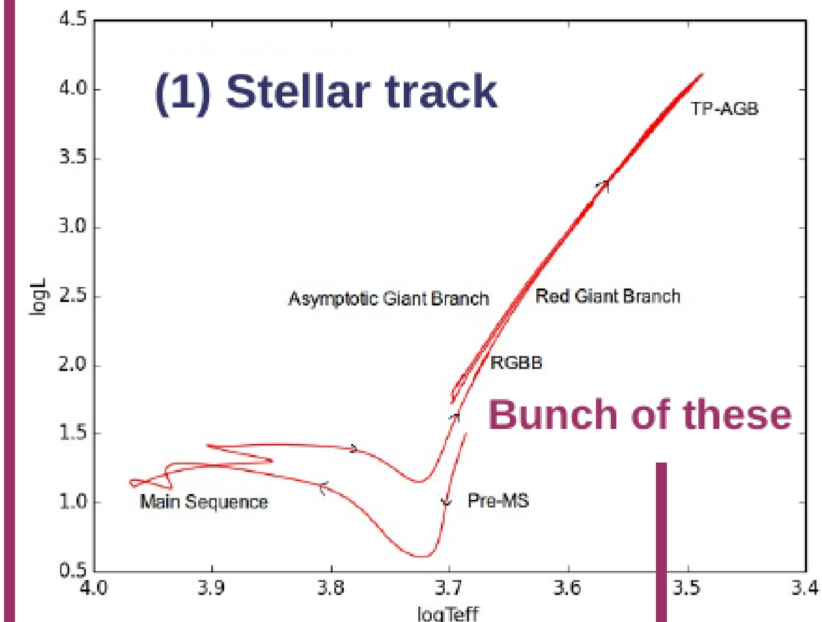
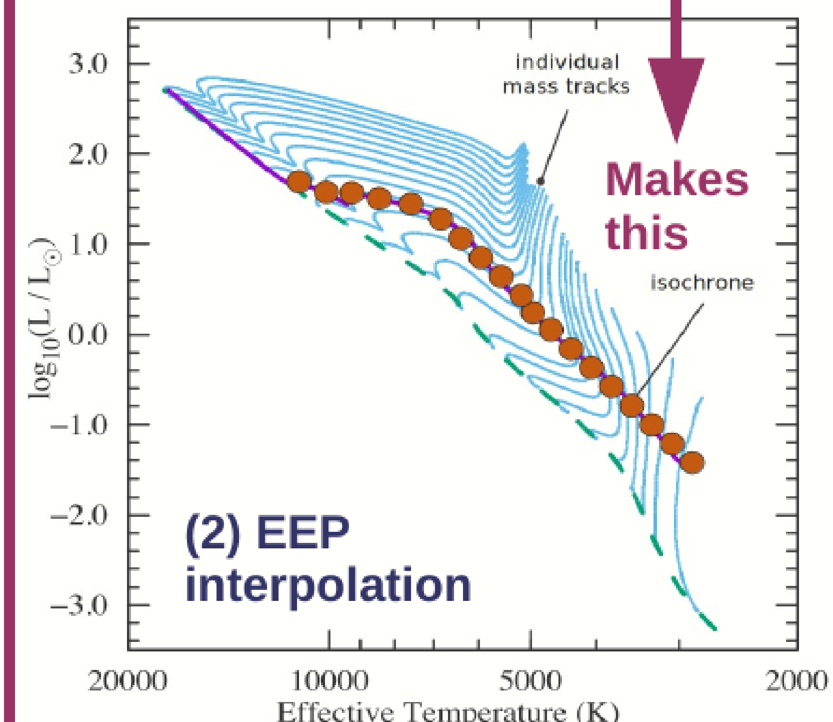
Isochrone review

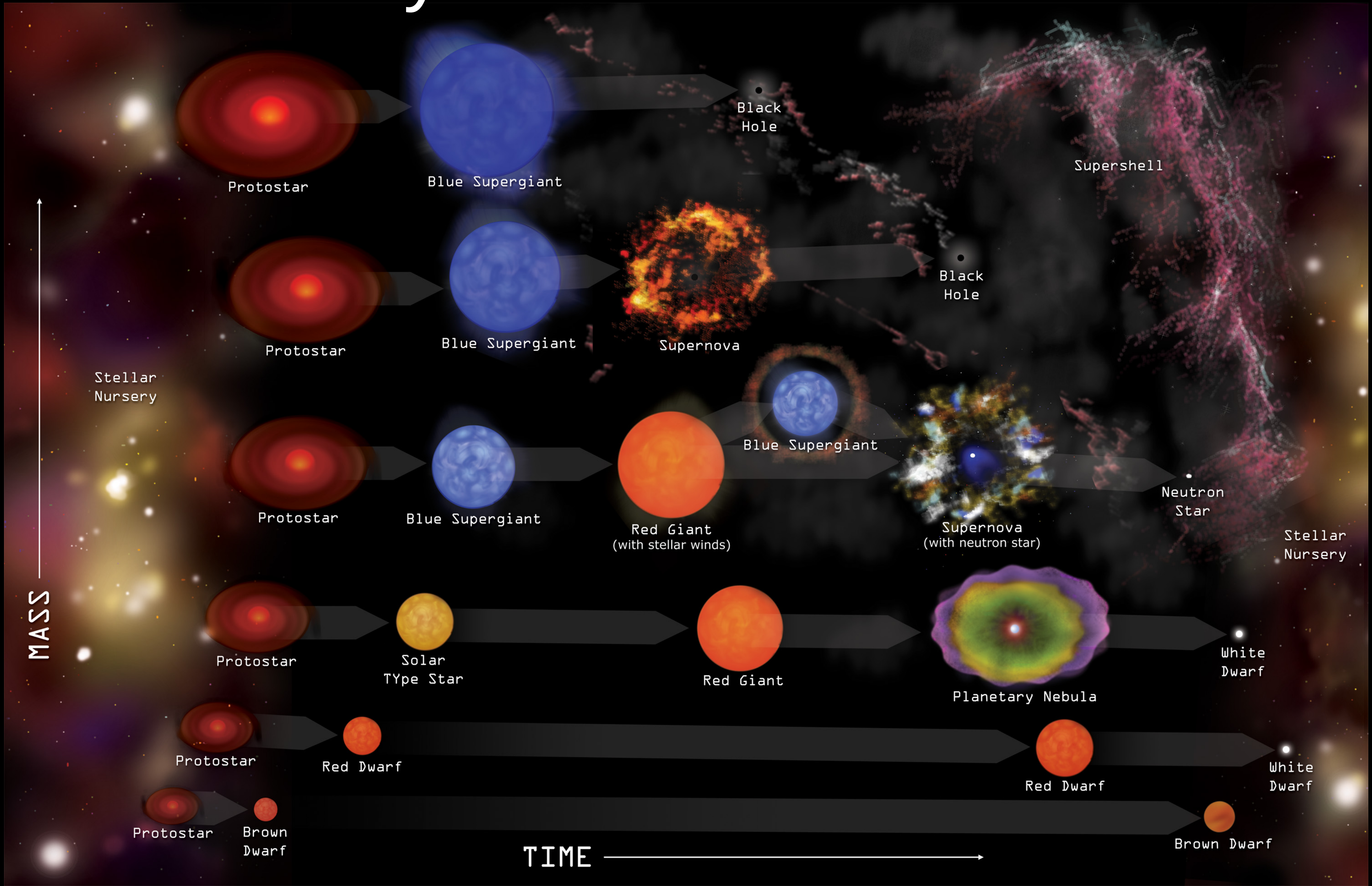






Derive fundamental parameters for both individual stars and stellar populations

→ Stellar modeling allows us to study how stars live and die



Our 10 billion dollar surveys are only as good as our interpretation of the data

Cosmic cliffs, JWST

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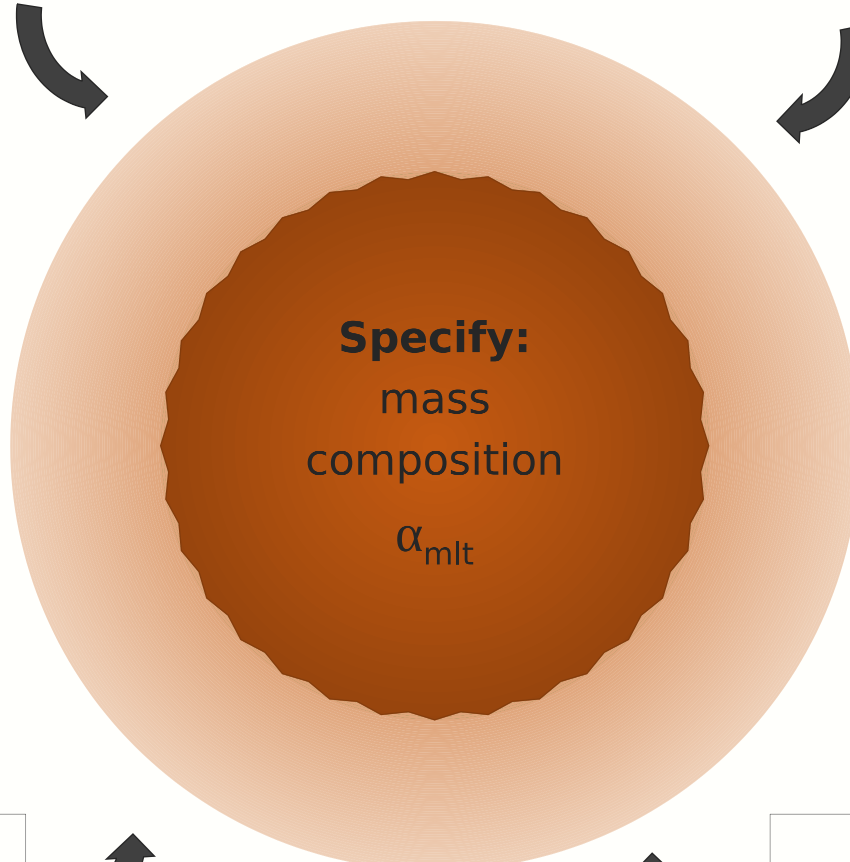
In the era of *Gaia*, TESS, SDSS, JWST, PLATO and any large survey, an essential goal is to estimate **non-observables** (such as **mass** and **age**) for huge numbers of stars → stellar models are how we obtain those non-observables

Components of a Stellar Structure & Evolution Program

Standard Equations:
mass & energy conservation
hydrostatic equilibrium
energy transport

Surface Boundary Conditions:
grey – Eddington $T(\tau)$
grey – Krishna Swamy $T(\tau)$
non-grey – stellar atmospheres

Microphysics:
opacities
equation of state
nuclear reactions



Properties:
radius
T_{eff}
luminosity
composition
P, T, ρ

Convection:
mixing length theory
full spectrum turbulence

Additional Physics:
diffusion
variable mass
rotation
magnetic fields / activity

Slide credit: Greg Feiden

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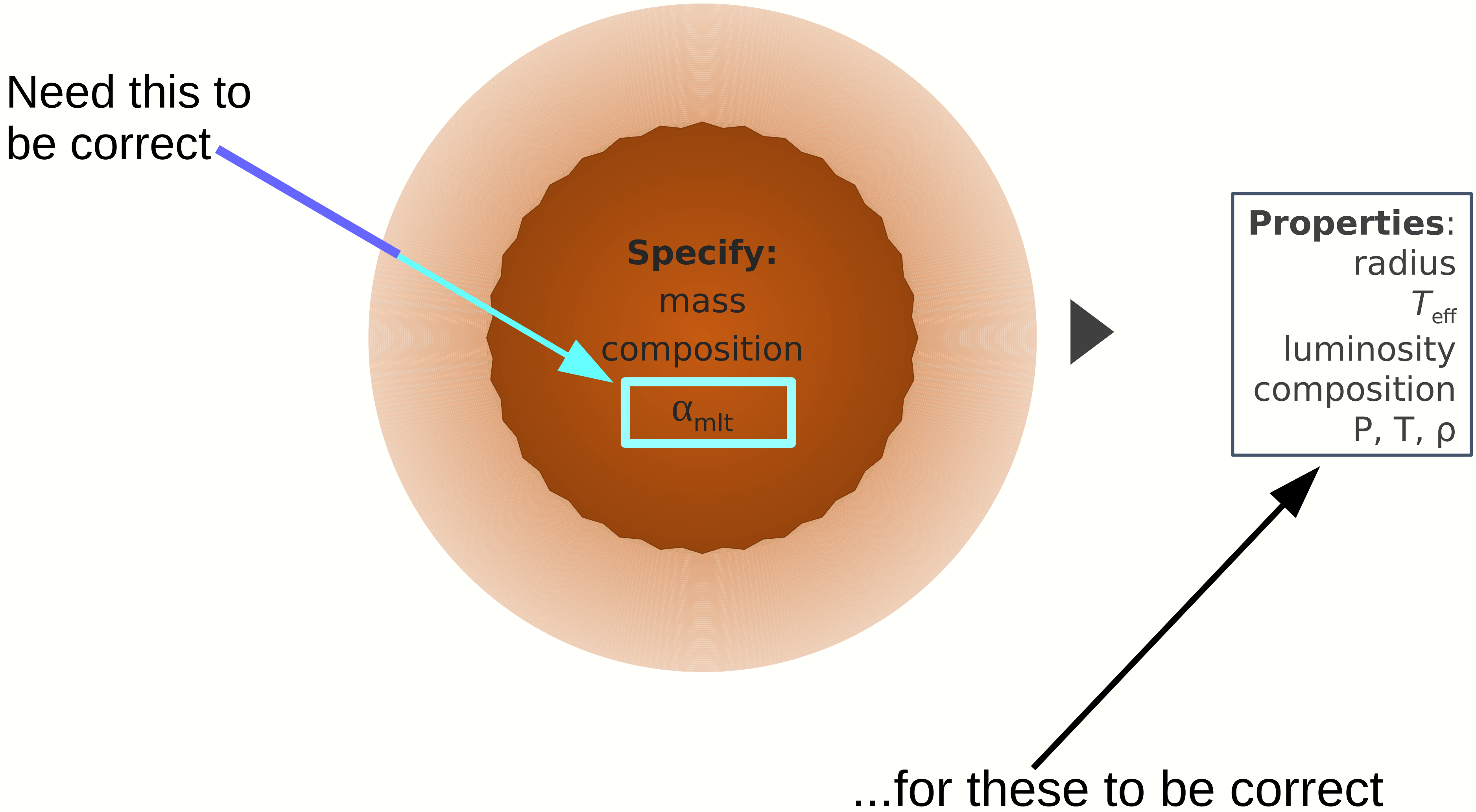
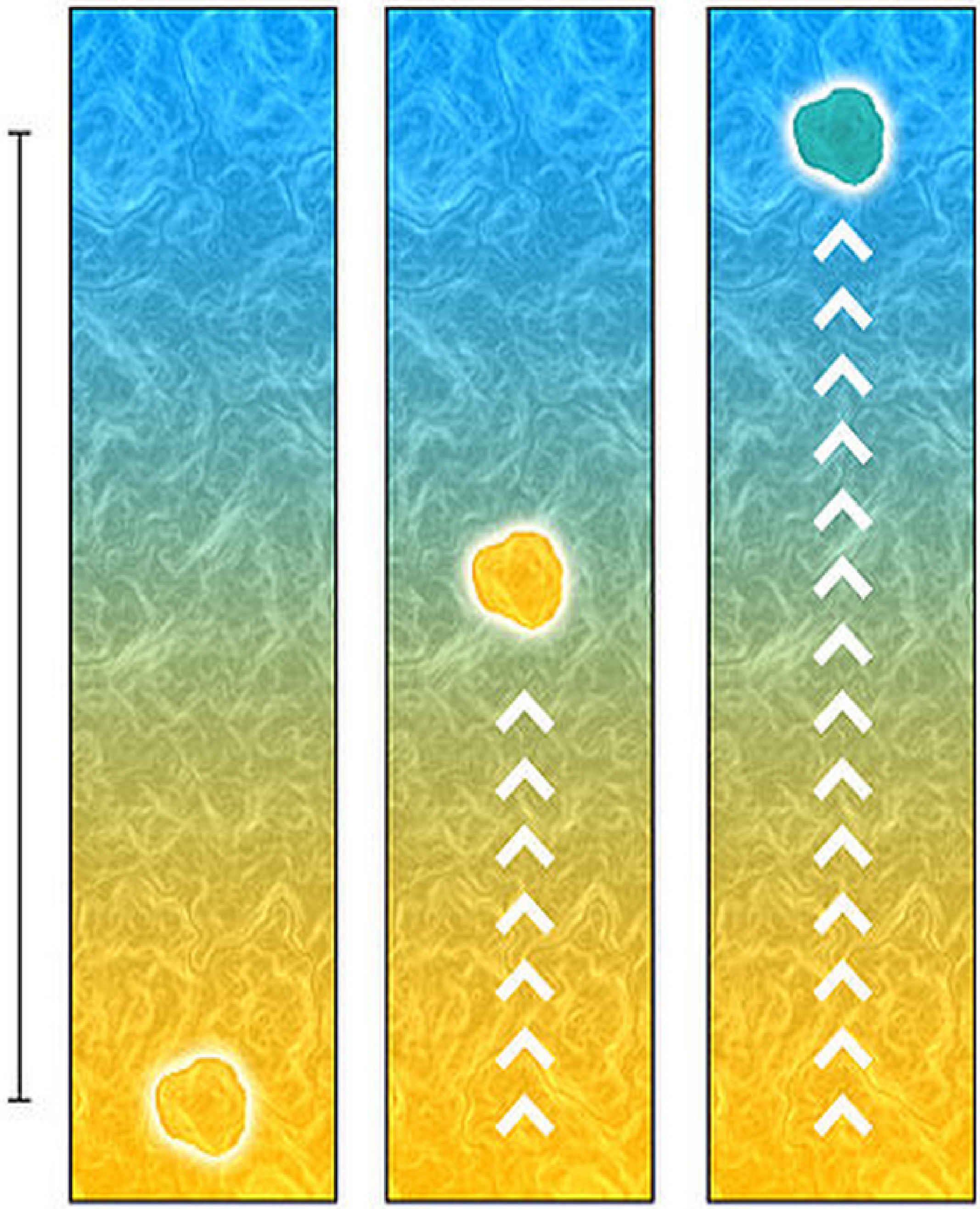


Mixing Length Theory (MLT) Formalism



$$F_{\rm conv} = \frac{1}{2}\rho v c_p T \frac{\lambda}{H_P}(\nabla_T - \nabla_{\rm ad}).$$

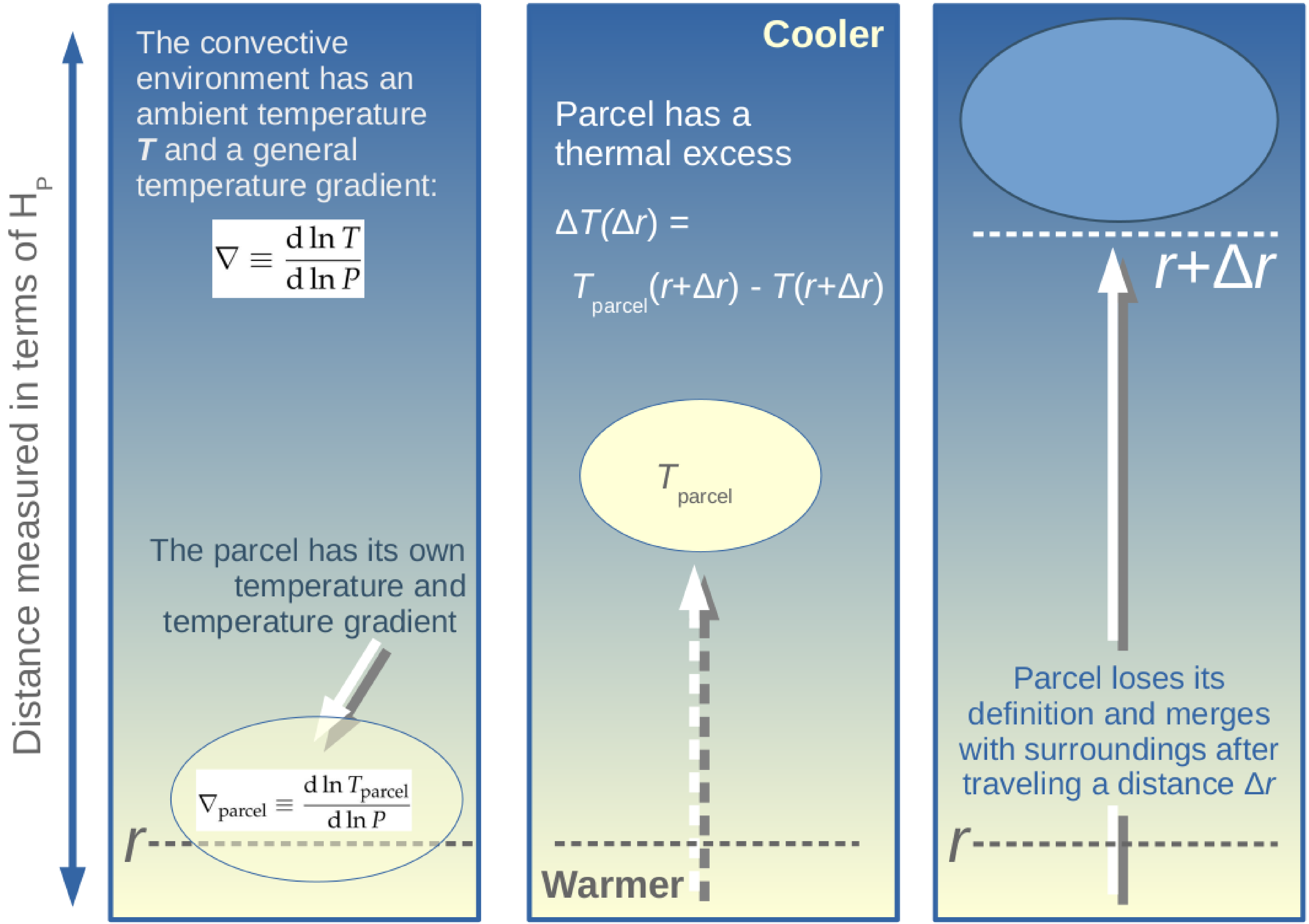
$$\alpha_{\rm MLT} = \frac{\lambda}{H_P} \qquad \nabla_T = \left(\frac{d\ln T}{d\ln P}\right)$$

-discrete parcels consisting of fluid with homogeneous properties are in pressure, but not thermal, equilibrium

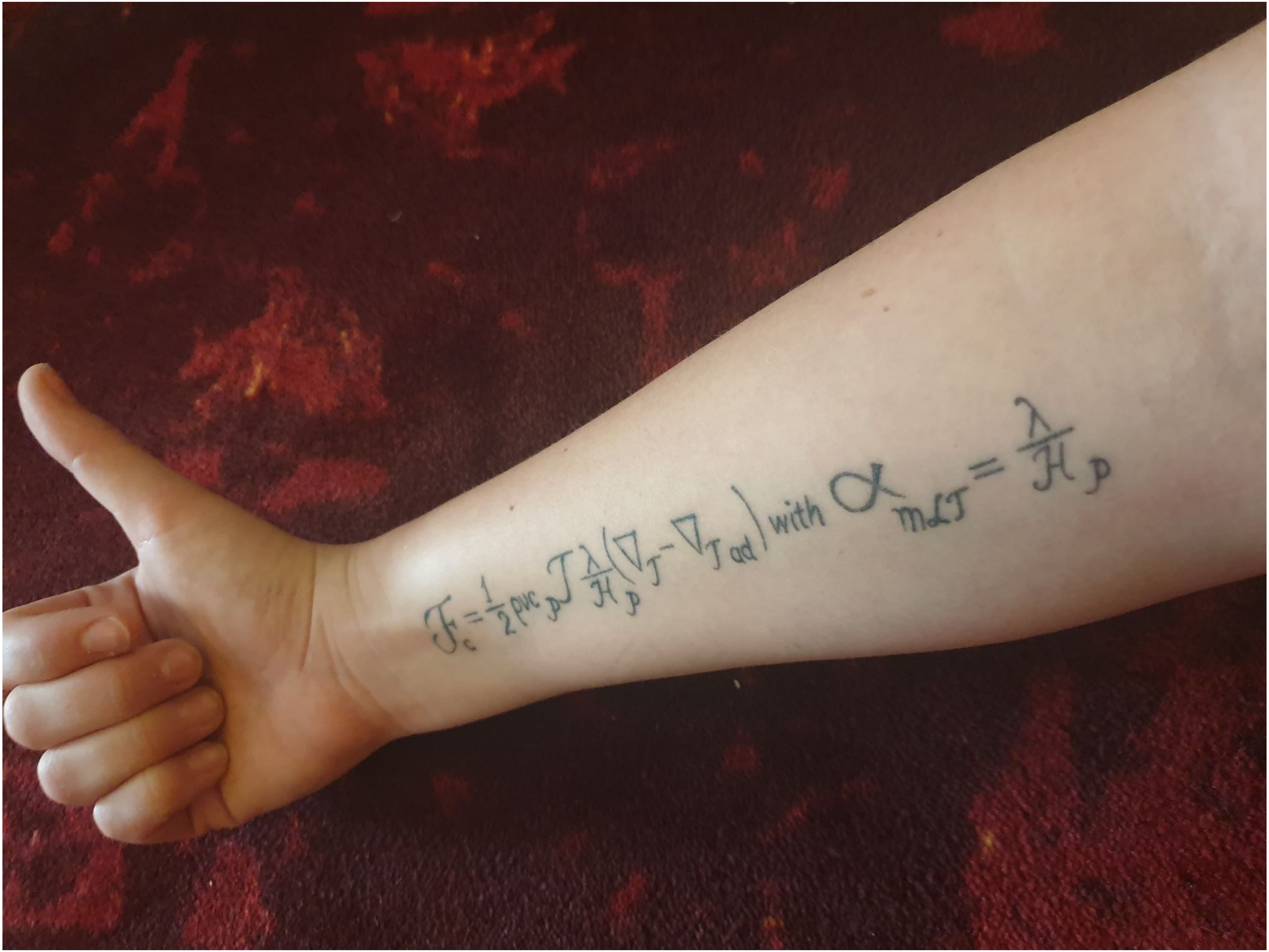
-parcels move along vertical trajectories

- “mixing length:” average distance which parcels can travel before denaturing

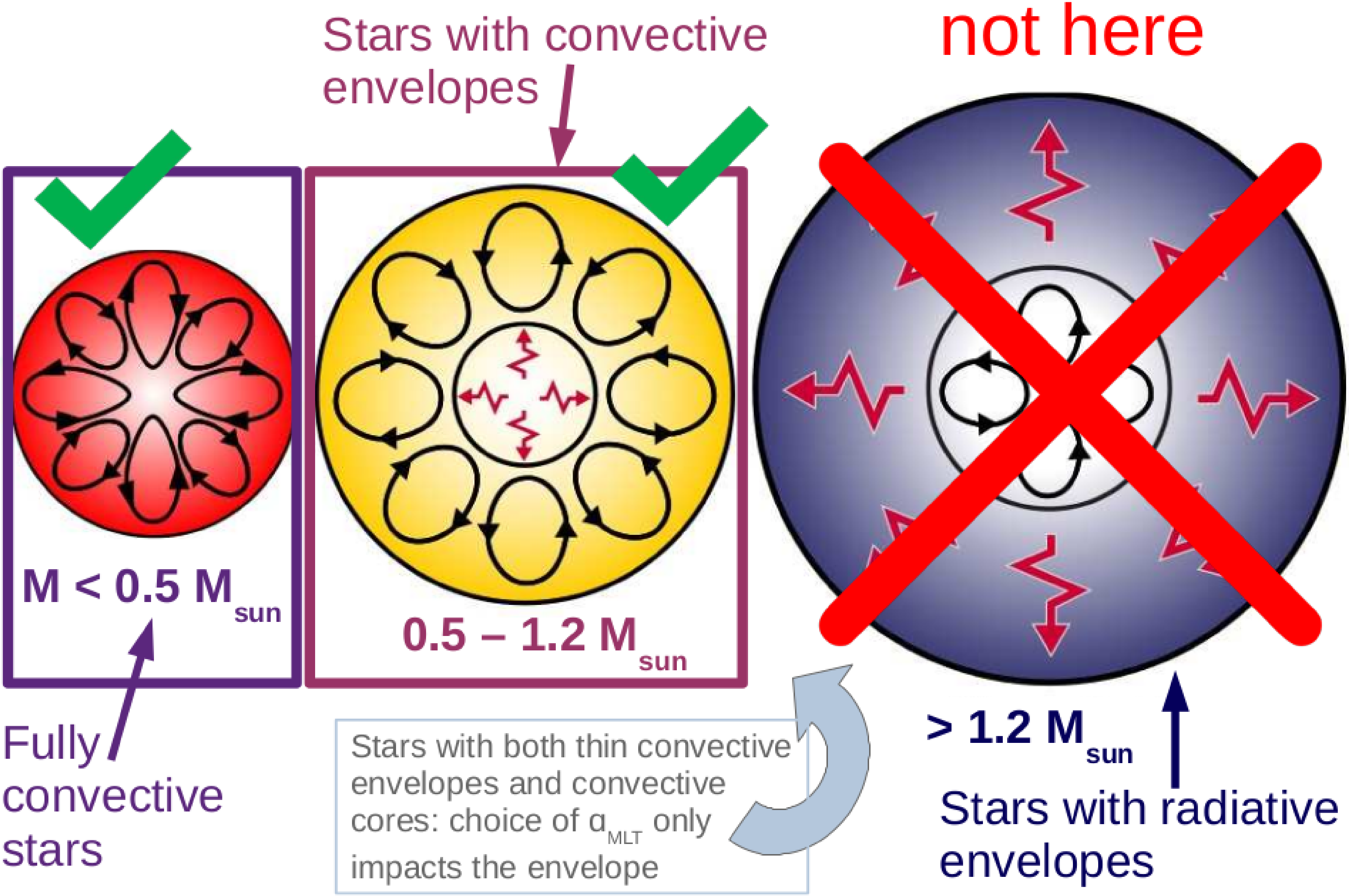
-α_{MLT} represents mean free path measured in pressure scale heights, $H_P = d\ln(P) / d\ln(T)$



from Joyce & Tayar Review, 2023


$F_c = \frac{1}{2} \rho v c_p T \frac{\lambda}{H_p} (\nabla_T - \nabla_{T\,ad})$ with $\alpha_{MLT} = \frac{\lambda}{H_p}$

Where the choice of α_{MLT} matters on the **main sequence**



from Joyce & Tayar Review, 2023

Check out my recent review on MLT with Jamie Tayar

galaxies

MDPI

Review

A Review of the Mixing Length Theory of Convection in 1D Stellar Modeling

Meridith Joyce [1,2,*,†] and Jamie Tayar [3,†]

1 Konkoly Observatory, Research Centre for Astronomy and Earth Sciences, Konkoly Th. M. út 15-17., H-1121 Budapest, Hungary
2 CSFK, MTA Centre of Excellence, Konkoly Thege Miklós út 15-17., H-1121 Budapest, Hungary
3 Department of Astronomy, University of Florida, Gainesville, FL 32611, USA; jtayar@ufl.edu
* Correspondence: meridith.joyce@csfk.org
† These authors contributed equally to this work.

Review

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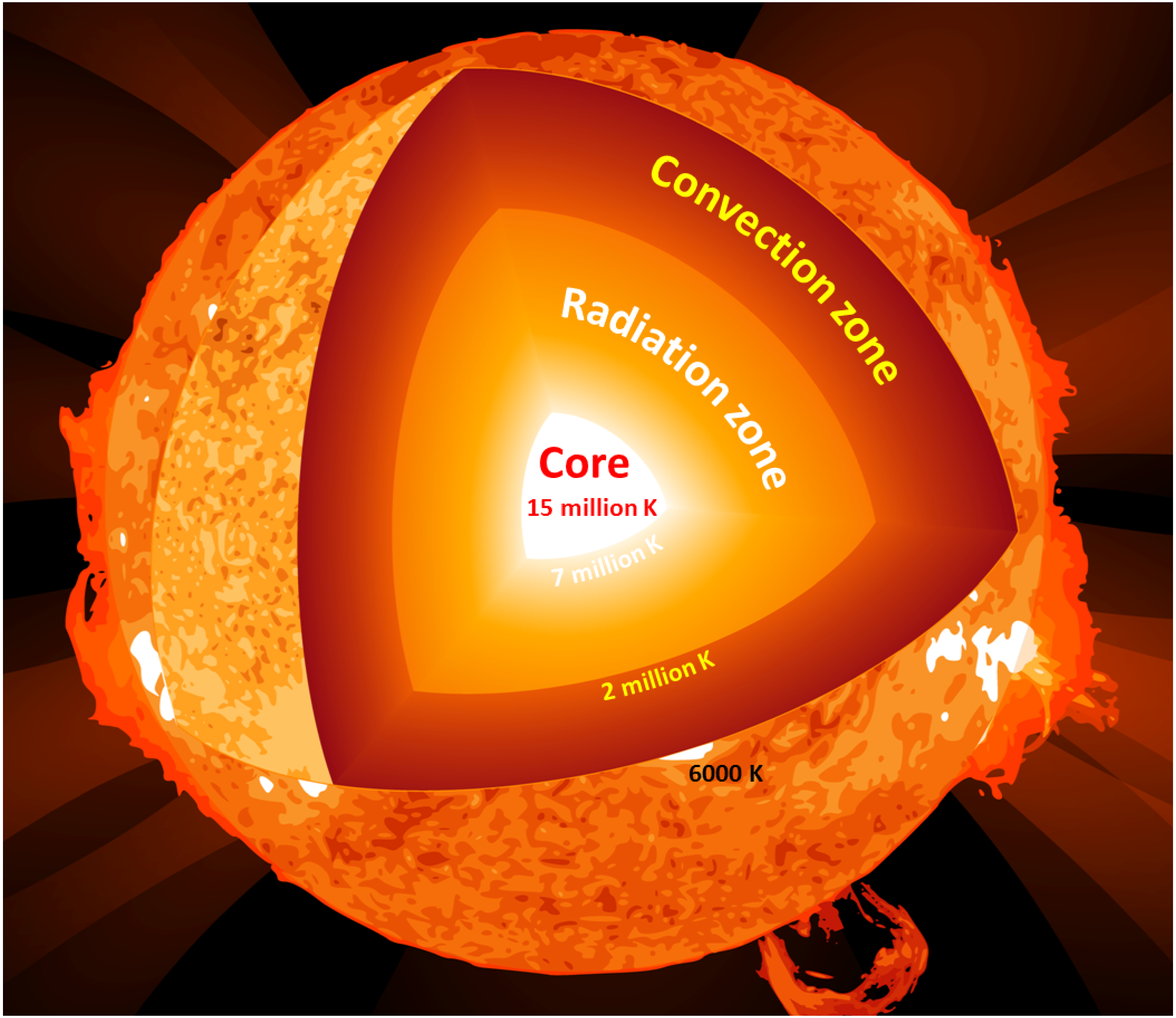
Special Issue
The Structure and Evolution of Stars
Edited by
Prof. Dr. Jorick Sandor Vink, Dr. Dominic Bowman and Dr. Jennifer Van Saders

MLT in practice: The solar calibration



MLT calibrations: the typical approach

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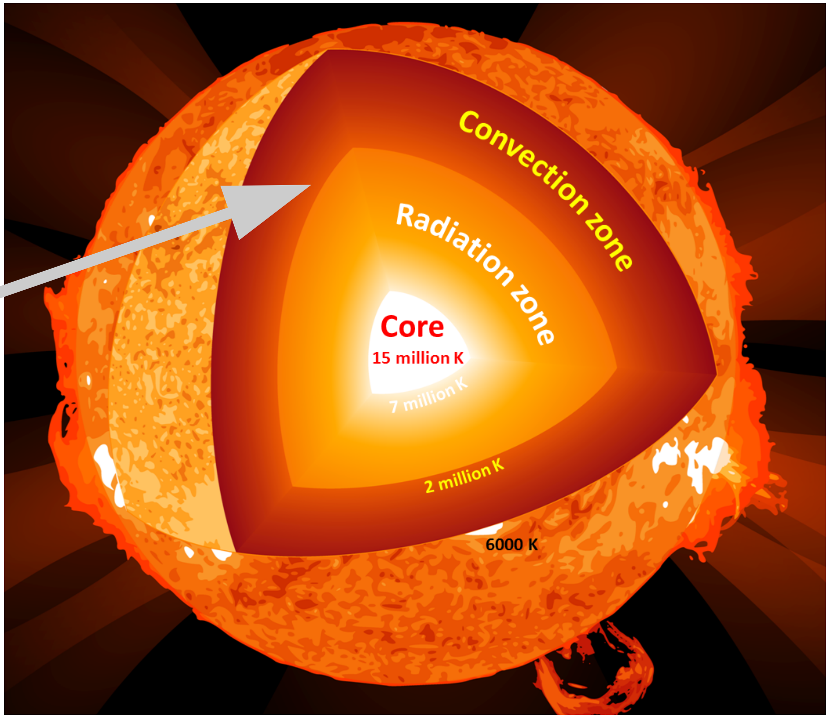
Obvious Problem: Not all stars are the Sun!

(Joyce & Chaboyer 2018a)

MLT calibrations
a pathway for mitigating modeling issues for **ideal** systems

Solution: Calibrate α_{MLT} to other stars, quantify the differences

Calibrate here:
- low mass stars (0.5 – 1.4 Ms)
- sub-surface convective envelope
- main sequence, subgiant, or early RGB



Two separate science questions:

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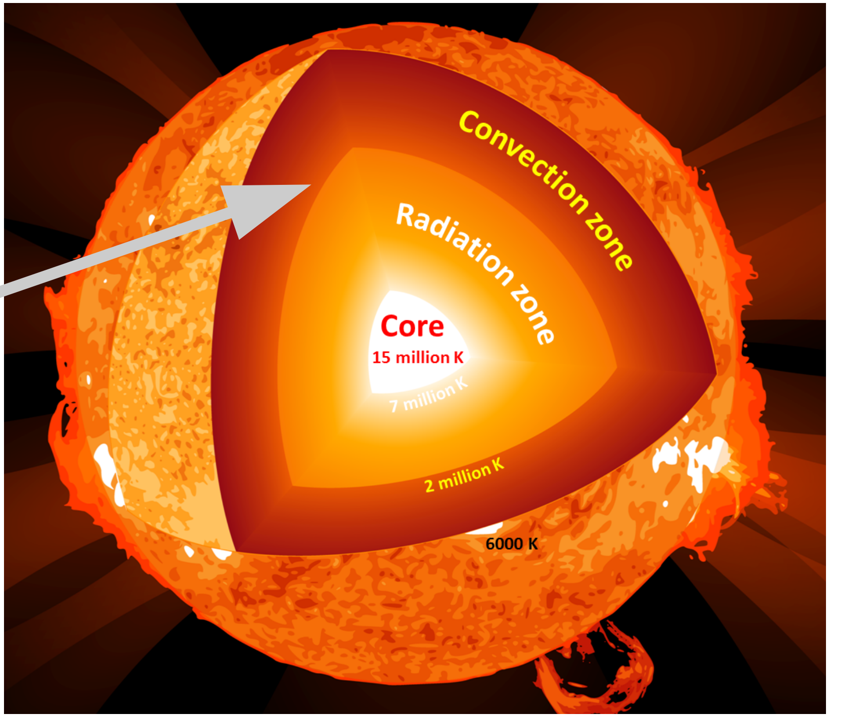

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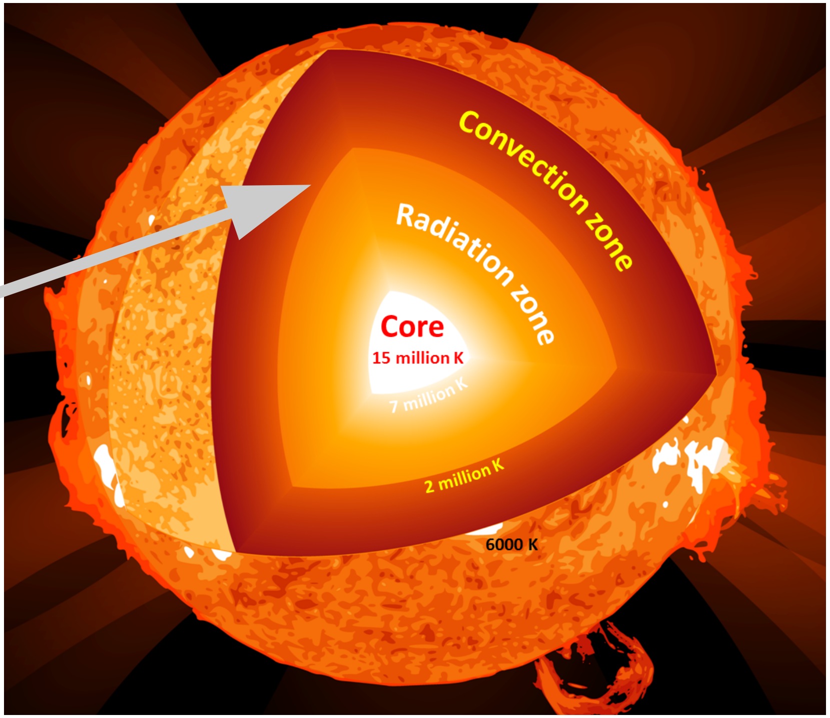

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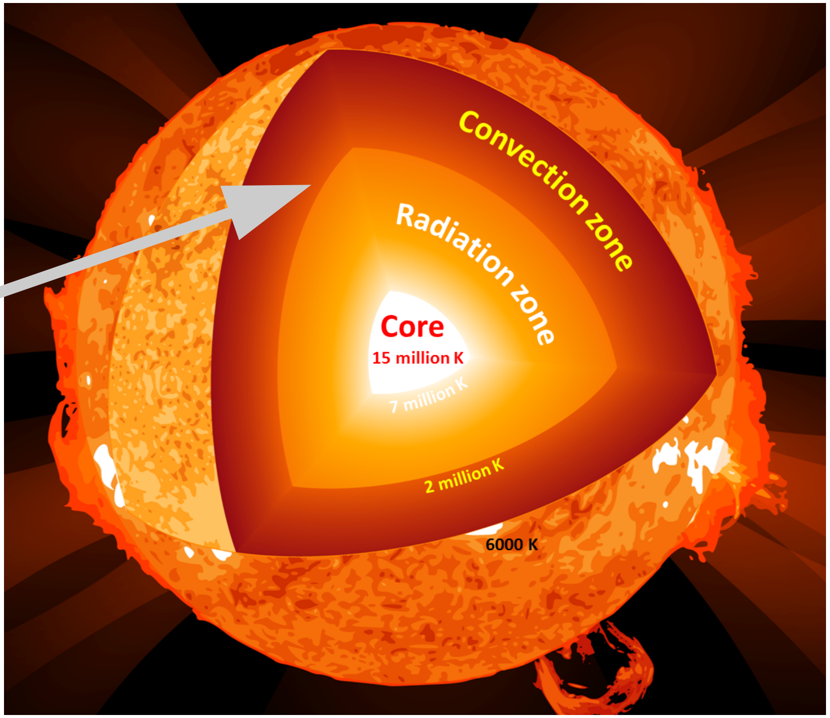

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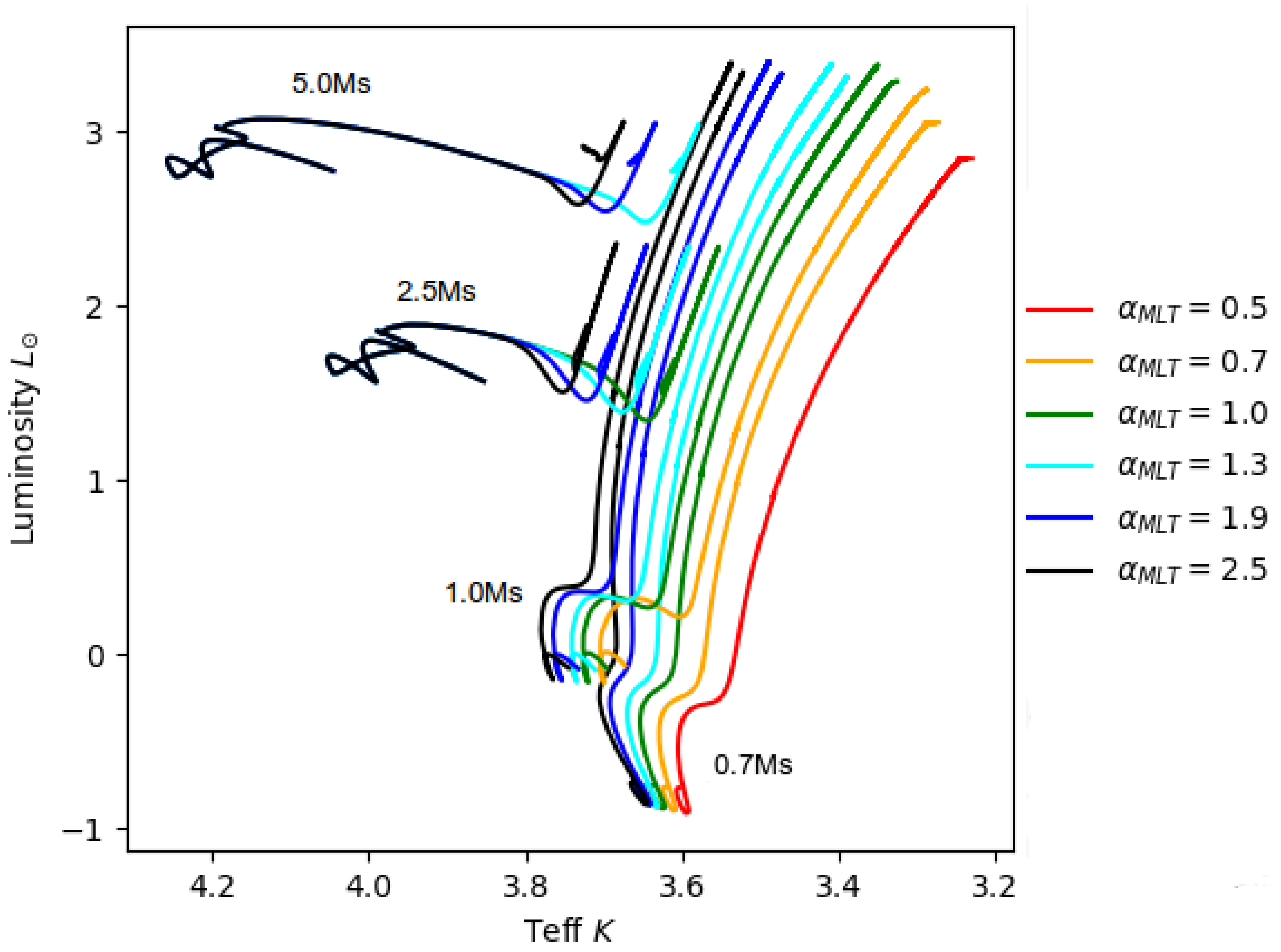
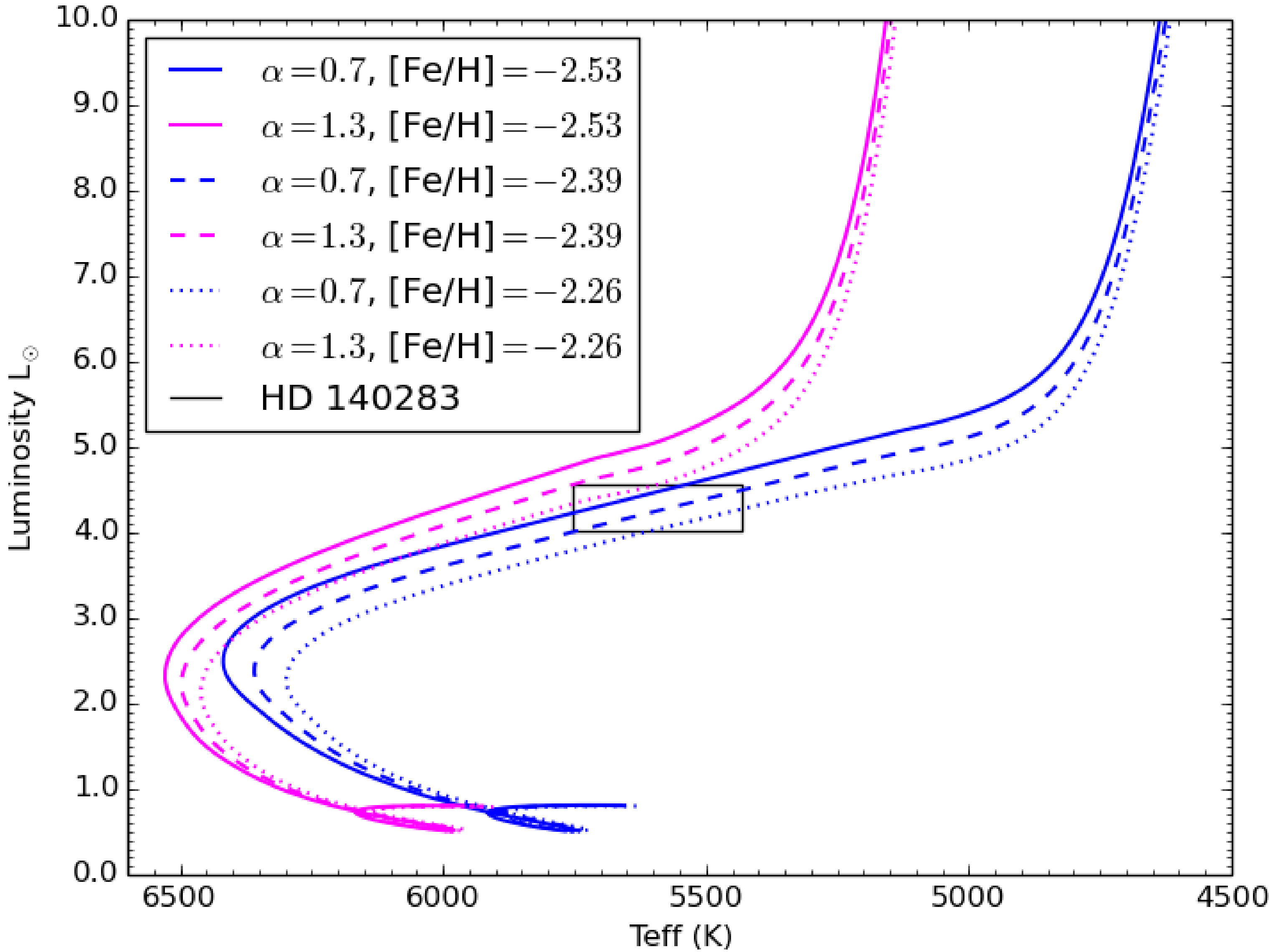
Example of mixing length's impact on low-intermediate mass stellar tracks



HD 140283: Can the notorious **mass—mixing length—metallicity degeneracy** be disentangled if a star is sufficiently well constrained and in the right part of the HR diagram?



(Joyce & Chaboyer, 2018a)

Fitting the metal-poor globular cluster M92:
Changing the mixing length in constituent tracks deeply affects the structure of an isochrone model

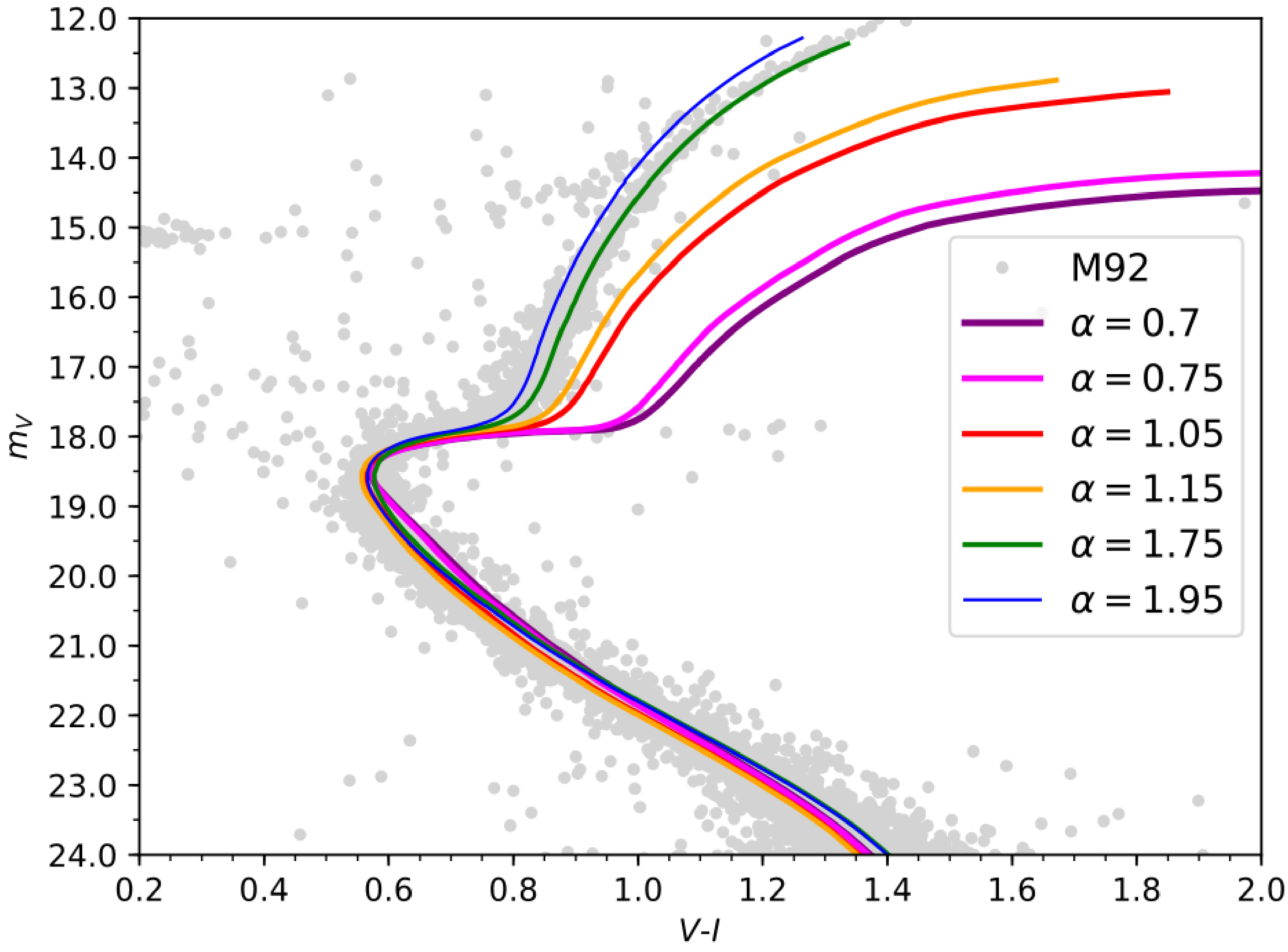
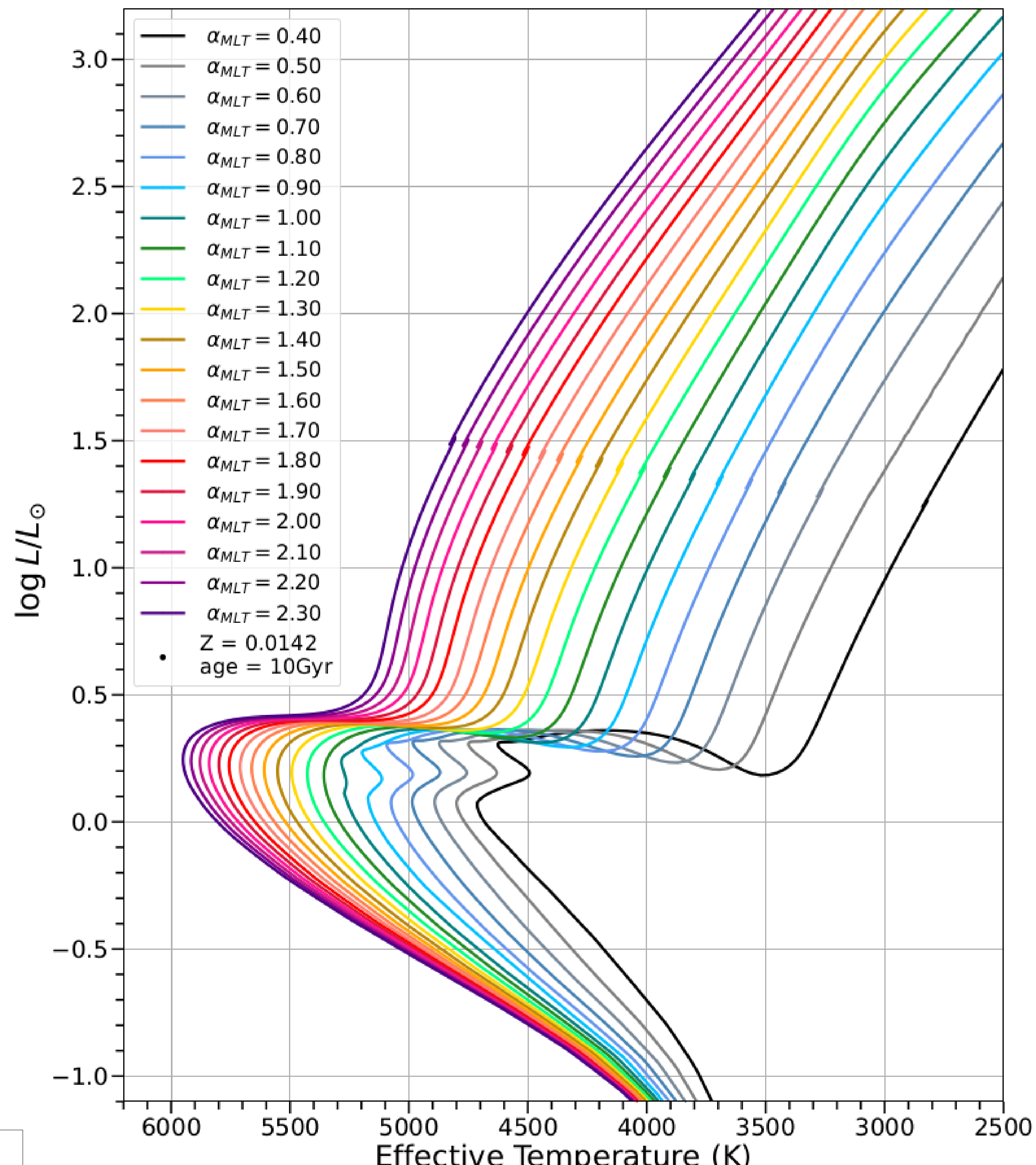


Figure 5. Six isochrones, each of age 13 Gyr, generated with different mixing lengths and shown against M92 for reference. Each isochrone in the figure

(Joyce & Chaboyer, 2018a)

Two important points:

The **precision of model-derived fundamental stellar parameters** is often **drastically overstated;** not enough effort is put toward quantifying the effects of parameter assumptions



from Joyce & Tayar Review, 2023

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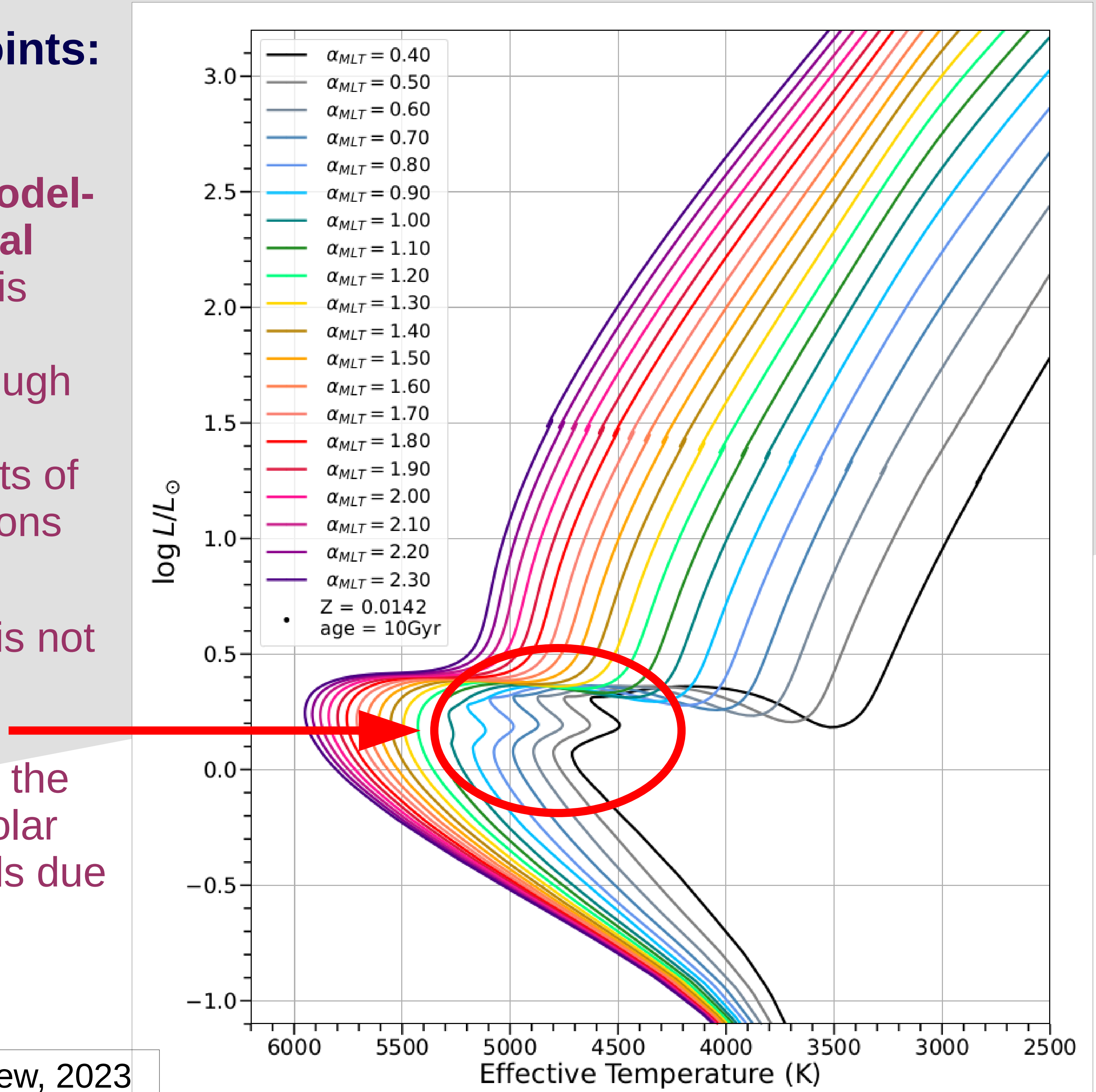
Impact on structure is not trivial! Note the **development of a convective core** on the main sequence of solar mass, solar-Z models due to small α_{MLT} values



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— so yes, of course it is wrong...how can we do better?

Using seismic parameters to calibrate the convective mixing length in highly constrained systems

α Centauri A & B

Classically and Asteroseismically Constrained 1D Stellar Evolution Models of Alpha Centauri A and B Using Empirical Mixing Length Calibrations

Meridith Joyce & Brian Chaboyer
ApJ, 2018

This study follows from the foundational work in the early 2000s to 2010s laid by many of the people in this room

- Frédéric Thévenin
- Lionel Bigot
- Pierre Kervella
- Michaël Bazot

in particular, the combination of interferometry with asteroseismic constraints

What makes alpha Cen the perfect lab for stellar modeling?

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In principle, the system is **over**-constrained, so finding a solution to our stellar modeling problem is possible but not guaranteed

The Ultimate Goal

Recall question 1

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(2) much more careful modeling & thoughtful statistics

α Centauri A & B: the next best thing to the Sun

Property	α Cen A	α Cen B	Reference
Mass $M_\odot$	1.1055 ± 0.004	0.9373 ± 0.003	Kervella et al. (2017)
Radius $R_\odot$	1.2234 ± 0.0053	0.8632 ± 0.004	Kervella et al. (2017)
Luminosity $L_\odot$	1.521 ± 0.015	0.503 ± 0.007	Kervella et al. (2017)
Z/X	0.039 ± 0.006	0.039 ± 0.006	Porto de Mello et al. (2008); Thoul et al. (2003)
Δ_1	105.9 ± 0.3	160.1 ± 0.1	de Meulenaer et al. (2010); Kjeldsen et al. (2005)
d_{02}	5.8 ± 0.1	10.7 ± 0.6	de Meulenaer et al. (2010); Kjeldsen et al. (2005)
r_{02}	0.055 ± 0.001	0.066 ± 0.004	de Meulenaer et al. (2010); Kjeldsen et al. (2005)

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- treat mass as fixed but flexible within the 1 sigma uncertainties

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r_{02}	0.055 ± 0.001	0.066 ± 0.004	de Meulenaer et al. (2010); Kjeldsen et al. (2005)

- we evolve large grids of single-star evolutionary tracks for each of alpha Cen A and alpha Cen B separately: valid approximation because they do not interact

- treat mass as fixed but flexible within the 1 sigma uncertainties

- assume they are the same age within 0.1 Myr. Time steps on the main sequence are ~10Myr, so pairs of tracks must be the same age within half a time step

α Centauri A & B: the next best thing to the Sun

Property	α Cen A	α Cen B	Reference
Mass $M_\odot$	1.1055 ± 0.004	0.9373 ± 0.003	Kervella et al. (2017)
Radius $R_\odot$	1.2234 ± 0.0053	0.8632 ± 0.004	Kervella et al. (2017)
Luminosity $L_\odot$	1.521 ± 0.015	0.503 ± 0.007	Kervella et al. (2017)
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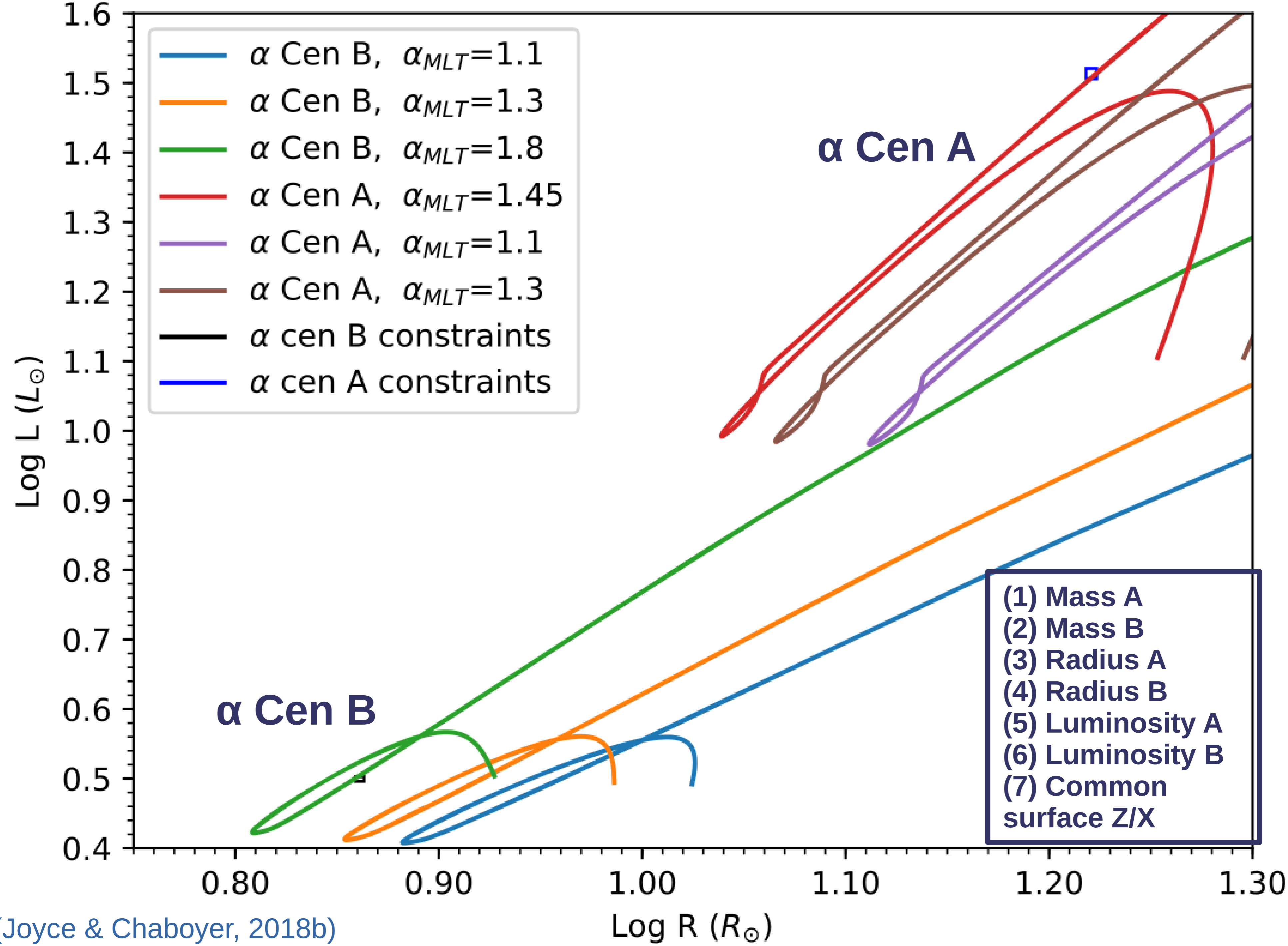
- the parameters left to vary freely are the initial helium (Y) and Z abundances and **the convective mixing length**

Classical optimization to α Centauri A & B



(Joyce & Chaboyer, 2018b)

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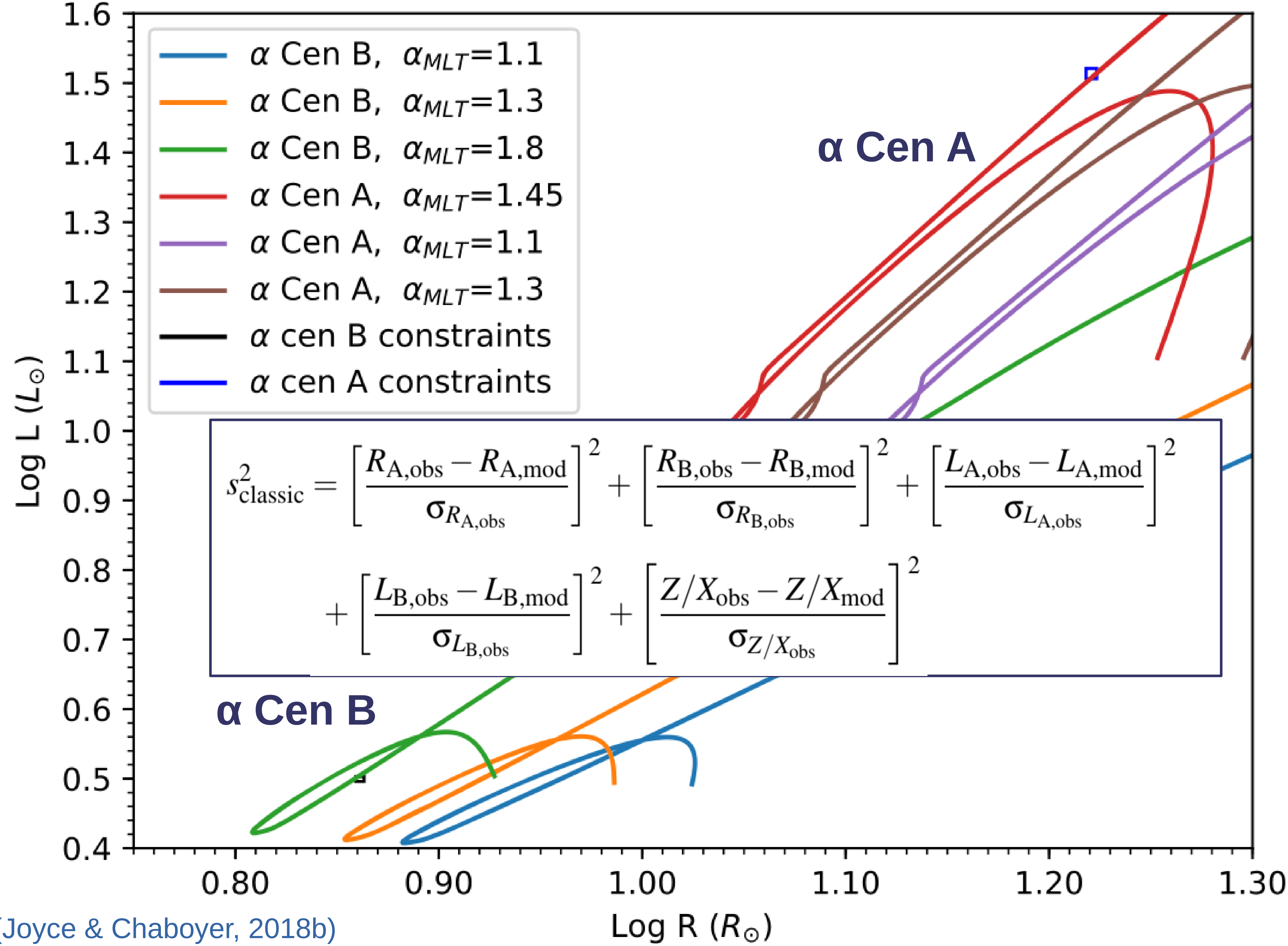
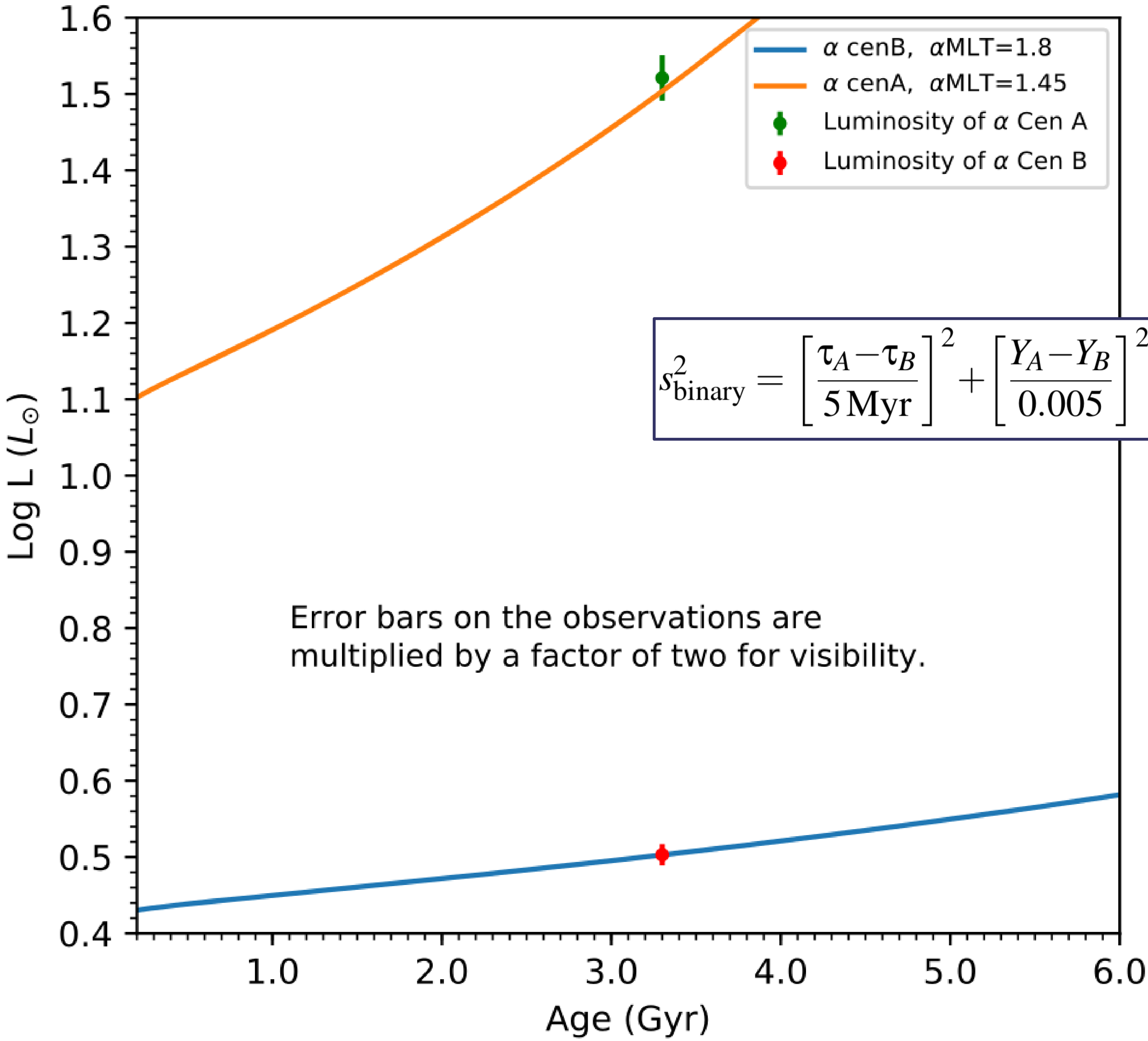


$$s^2_{\text{classic}} = \left[\frac{R_{A,\text{obs}} - R_{A,\text{mod}}}{\sigma_{R_{A,\text{obs}}}}\right]^2 + \left[\frac{R_{B,\text{obs}} - R_{B,\text{mod}}}{\sigma_{R_{B,\text{obs}}}}\right]^2 + \left[\frac{L_{A,\text{obs}} - L_{A,\text{mod}}}{\sigma_{L_{A,\text{obs}}}}\right]^2 + \left[\frac{L_{B,\text{obs}} - L_{B,\text{mod}}}{\sigma_{L_{B,\text{obs}}}}\right]^2 + \left[\frac{Z/X_{\text{obs}} - Z/X_{\text{mod}}}{\sigma_{Z/X_{\text{obs}}}}\right]^2$$

(Joyce & Chaboyer, 2018b)

The condition of simultaneity



$$s^2_{\mathrm{binary}} = \left[\frac{\tau_A - \tau_B}{5\,\mathrm{Myr}}\right]^2 + \left[\frac{Y_A - Y_B}{0.005}\right]^2 + \left[\frac{Z_A - Z_B}{0.0005}\right]^2$$

Ensuring robustness across physical prescriptions

Standard: Eddington approximation; grey model atmosphere

KS: Krishna Swamy approximation; grey model atmosphere

Low diffusion: coefficient eta describing the diffusion of heavy elements (diffused as iron) in the outer layers is set to half of its default efficiency

High diffusion: eta is set to 1.5x its default efficiency

Overshoot: convective boundary mixing is permitted at 0.1x the pressure scale height

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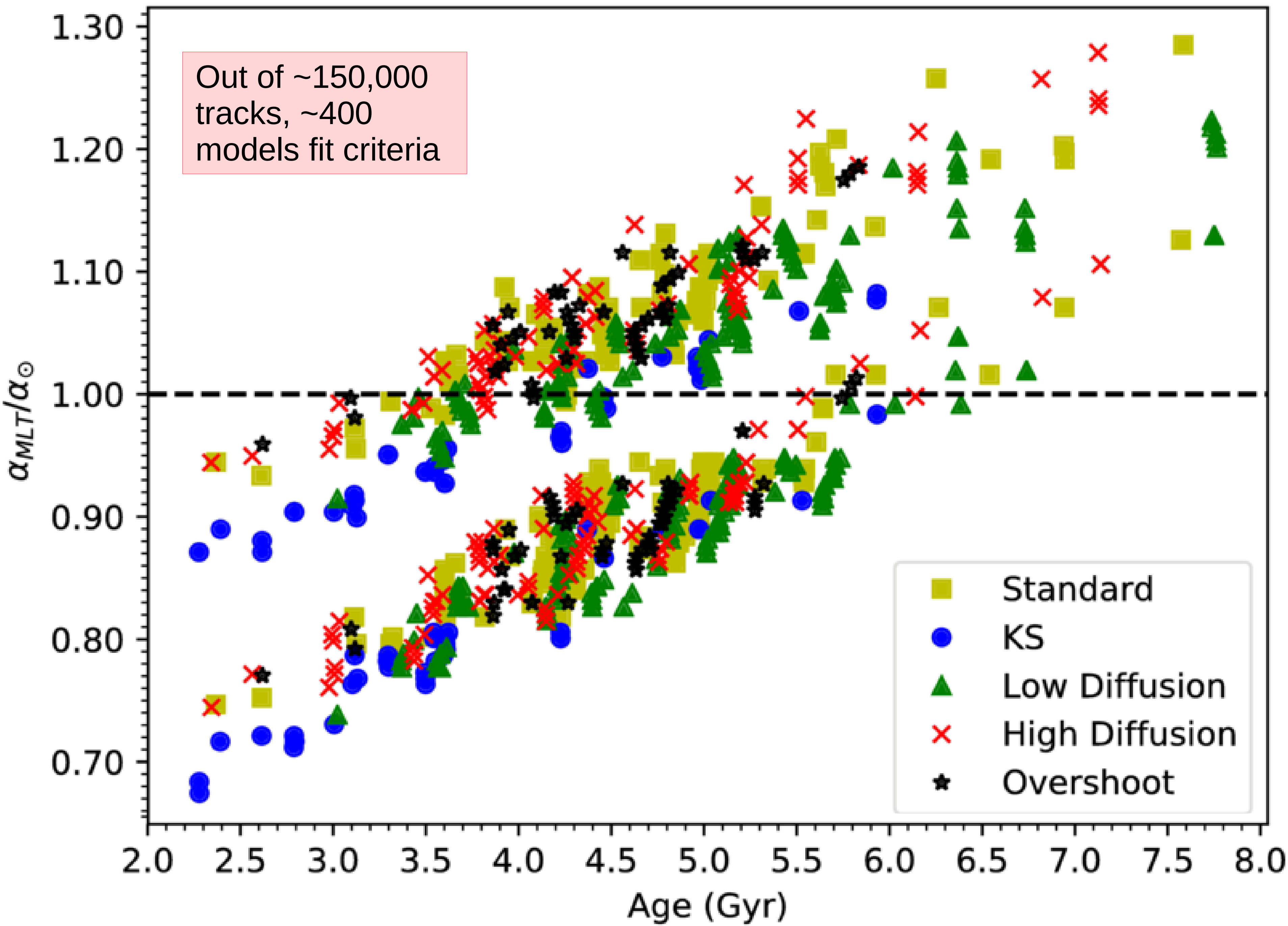
A solar calibration of alpha_MLT must be computed separately for each configuration so that the results can be compared self-consistently

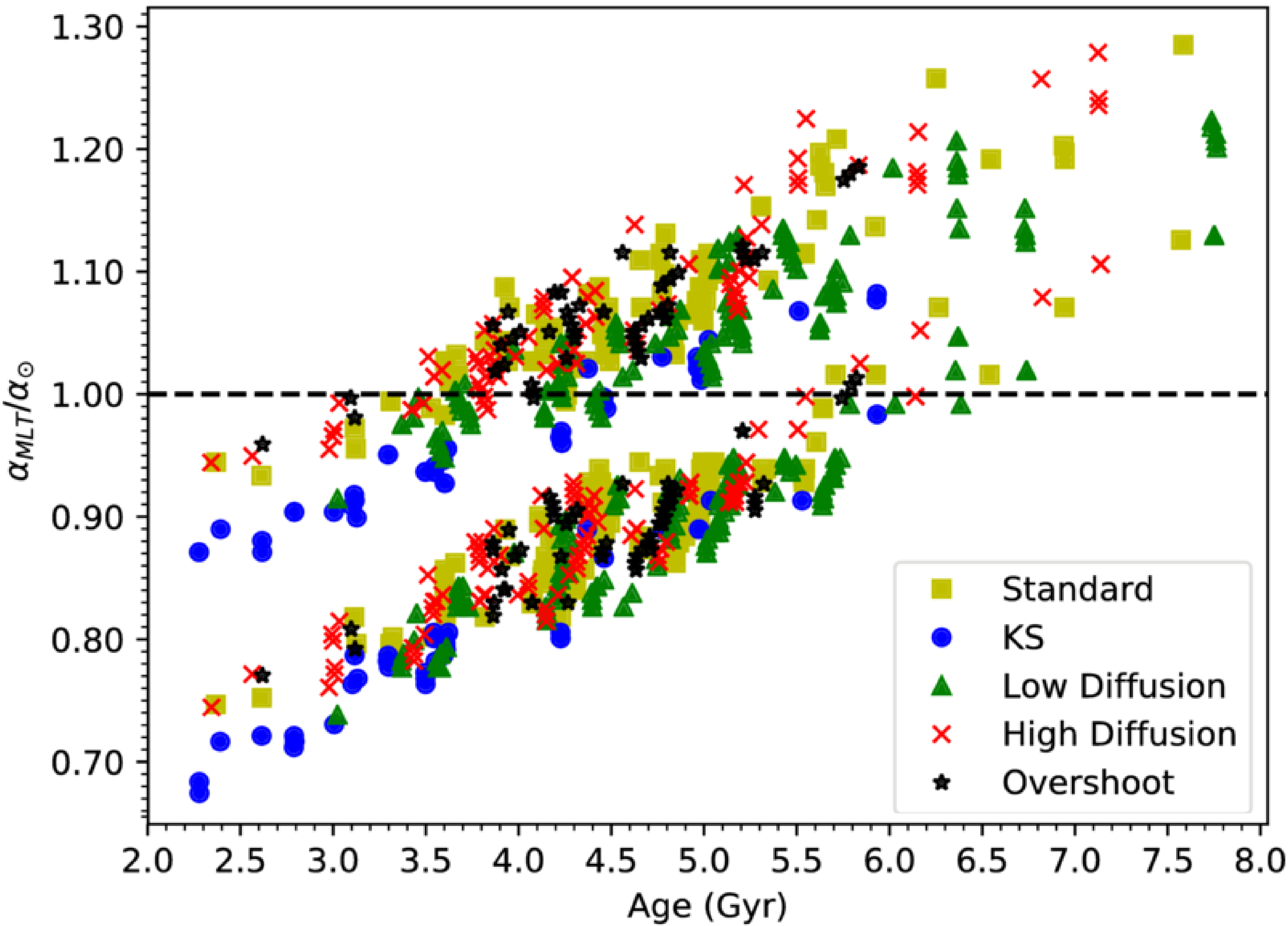
THEORETICAL PARAMETERS OF SOLAR-CALIBRATED MODELS

Config Name	Atmosphere	η_D	α_{ovs}	$\alpha_\odot$	Y_{in}	Z_{in}	$\Delta\nu_{n,1}$	$\delta\nu_{n,0}$	r_{02}
Standard	Eddington	1.0	0.0	1.8210	0.27	0.018	135.4	9.85	0.0728
KS	Krishna Swamy	1.0	0.0	2.1353	0.27	0.018	135.0	9.83	0.0728
Low Diffusion	Eddington	0.5	0.0	1.8148	0.28	0.020	135.6	9.89	0.0729
High Diffusion	Eddington	1.5	0.0	1.8535	0.27	0.018	134.6	9.67	0.0718
Overshoot	Eddington	1.5	0.1	1.8559	0.27	0.018	135.2	9.68	0.0716

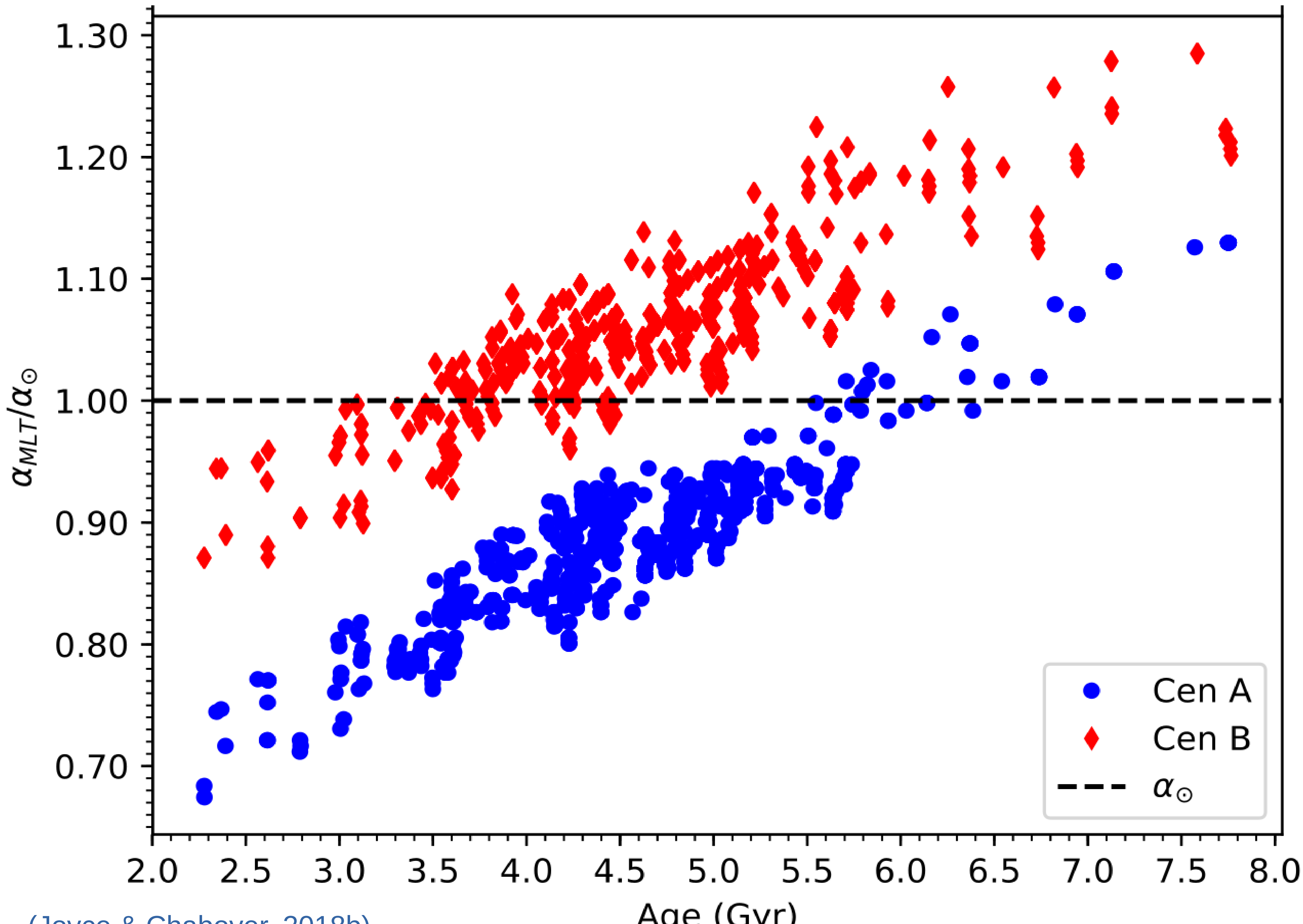
Each type of marker is a different physical prescription



Choice of input physics has some effect on fitted age

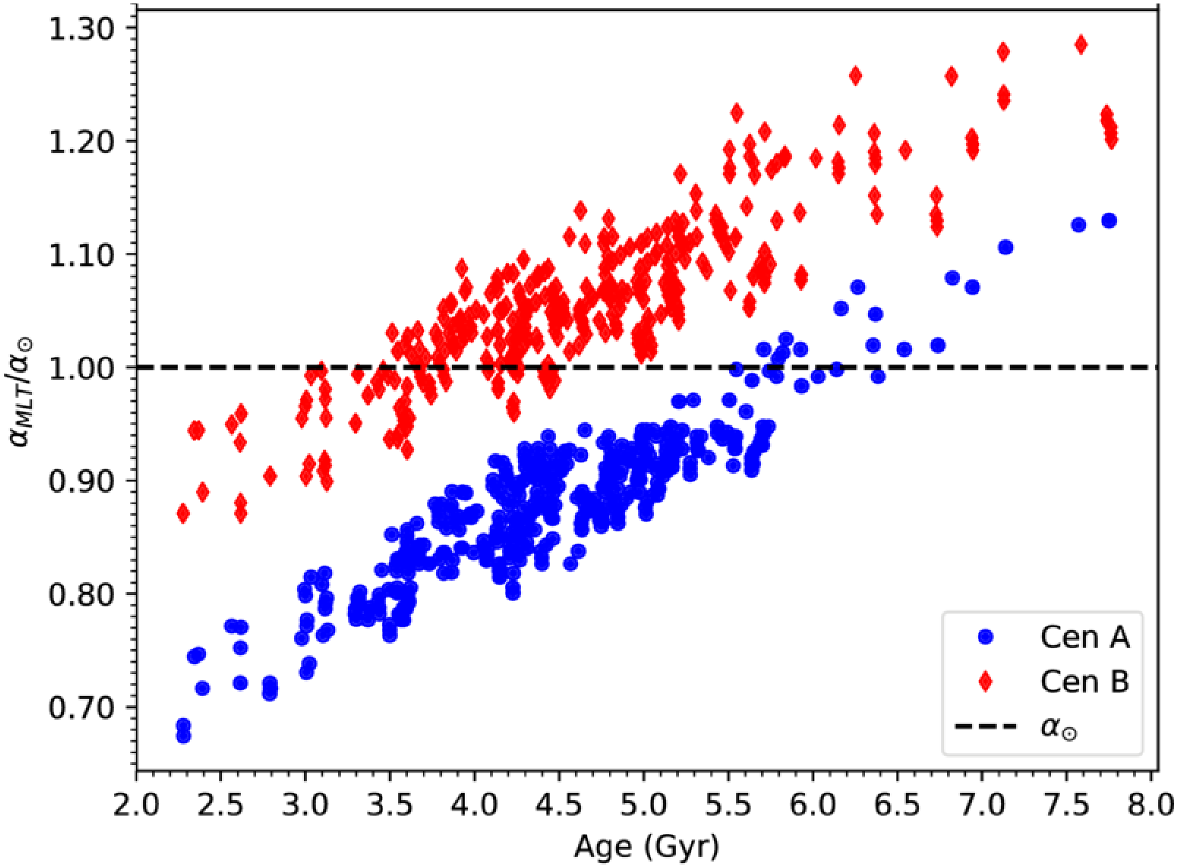


If we separate them by solar-normalized mixing length...



(Joyce & Chaboyer, 2018b)

Mixing length relation on alpha Centauri A & B: classical and binary constraints only



Using an agreement statistic comprising 7 classical conditions and a common age, we see a clear bifurcation in αMLT: it is always larger for α Cen B than for α Cen A *regardless of other input physics*

$$s^2_{\text{binary}} = \left[\frac{\tau_A - \tau_B}{5\,\text{Myr}}\right]^2 + \left[\frac{Y_A - Y_B}{0.005}\right]^2 + \left[\frac{Z_A - Z_B}{0.0005}\right]^2$$

$$s^2_{\text{classic}} = \left[\frac{R_{\text{A,obs}} - R_{\text{A,mod}}}{\sigma_{R_{\text{A,obs}}}}\right]^2 + \left[\frac{R_{\text{B,obs}} - R_{\text{B,mod}}}{\sigma_{R_{\text{B,obs}}}}\right]^2 + \left[\frac{L_{\text{A,obs}} - L_{\text{A,mod}}}{\sigma_{L_{\text{A,obs}}}}\right]^2$$

$$+ \left[\frac{L_{\text{B,obs}} - L_{\text{B,mod}}}{\sigma_{L_{\text{B,obs}}}}\right]^2 + \left[\frac{Z/X_{\text{obs}} - Z/X_{\text{mod}}}{\sigma_{Z/X_{\text{obs}}}}\right]^2$$

Parameter

Classical & Binary

$\alpha_{MLT, A}$	~0.7-1.1x solar value
$\alpha_{MLT, B}$	~0.9-1.3x solar value -always higher than Cen A's value within a given pair
Age	Anywhere from 2 to 8 Gyr, spanning most estimates in the literature from the past 20 years (i.e. not useful)

Incorporating seismic constraints

We would like an additional constraint on the stellar interior. Asteroseismic quantities can be useful as long as they are **not** impacted by the surface layers!

$$\Delta_l(n) = \nu_{n,l} - \nu_{n-1,l}, \quad (1)$$

$$d_{l,l+2}(n) = \nu_{n,l} - \nu_{n-1,l+2}, \quad (2)$$

$$r_{02}(n) = \frac{d_{0,2}(n)}{\Delta_1(n)}. \quad (3)$$

Roxburgh & Voronstov (2003)

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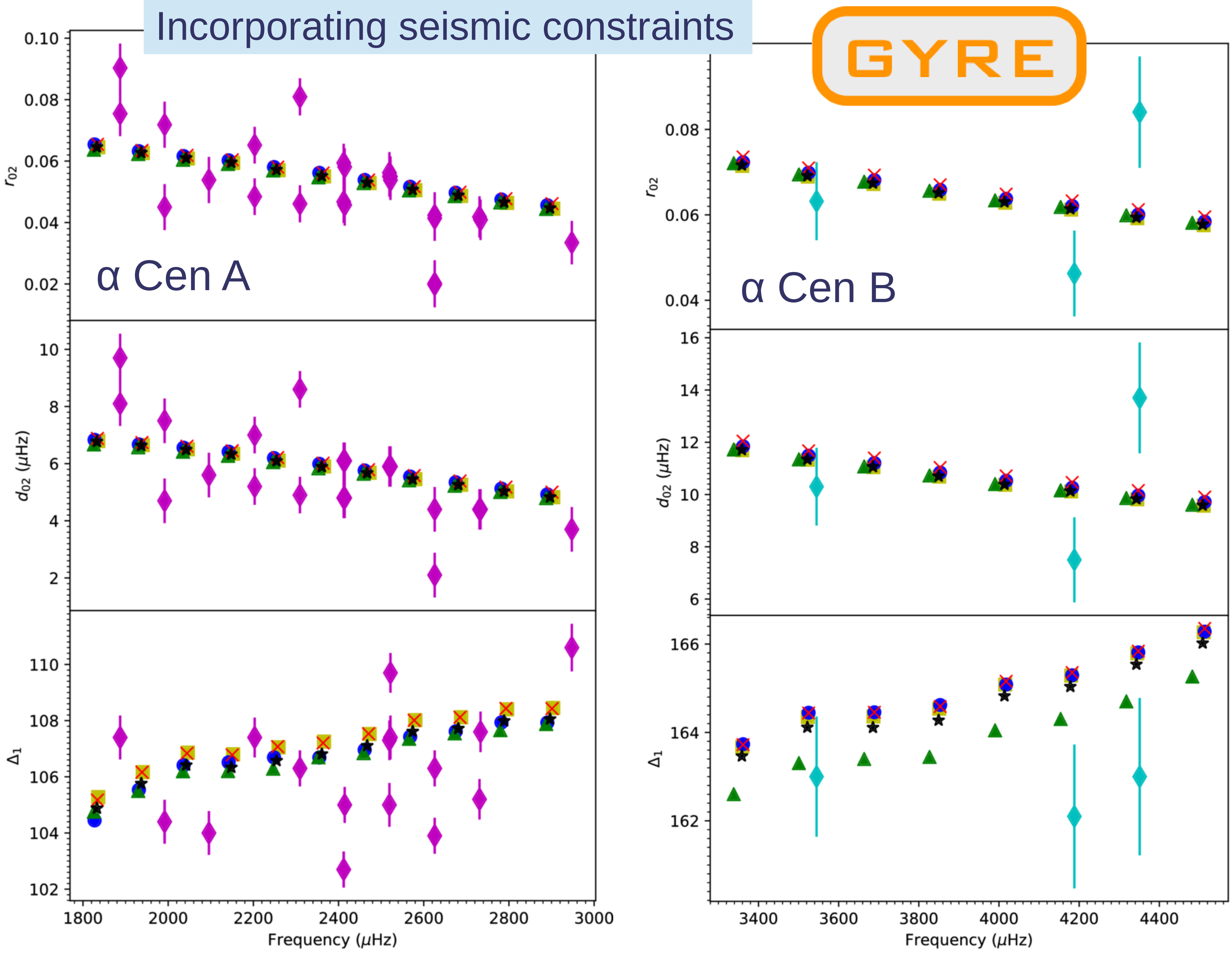
Roxburgh & Voronstov (2003)

Introducing the additional term:

$$s^2_{\text{seismic,w}} = \frac{1}{2}\left[\frac{r_{\text{A,obs}} - r_{\text{A,mod}}}{\sigma_{r_{02,\text{A}}}}\right]^2 + \left[\frac{r_{\text{B,obs}} - r_{\text{B,mod}}}{\sigma_{r_{02,\text{B}}}}\right]^2$$

Incorporating seismic constraints



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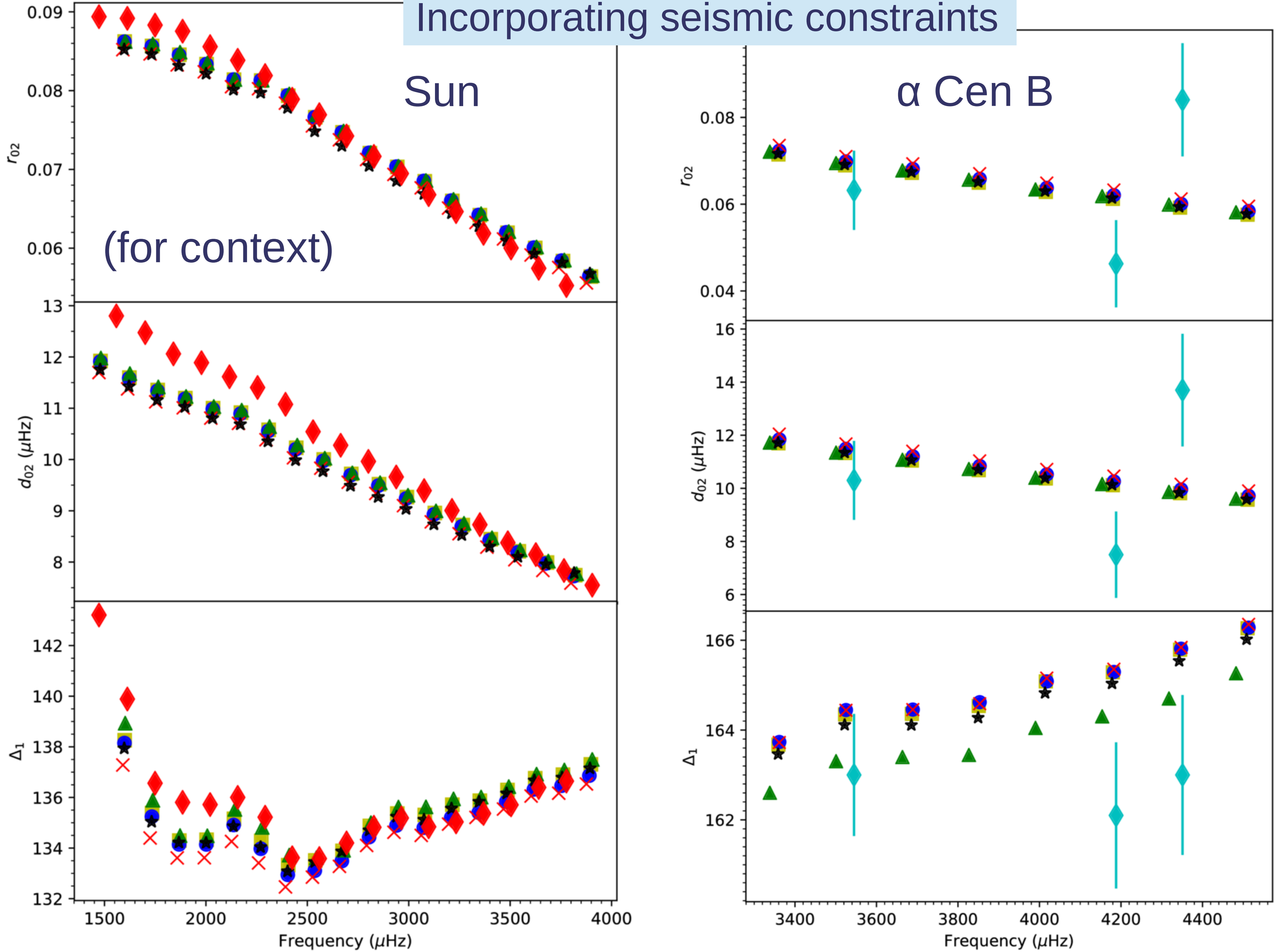
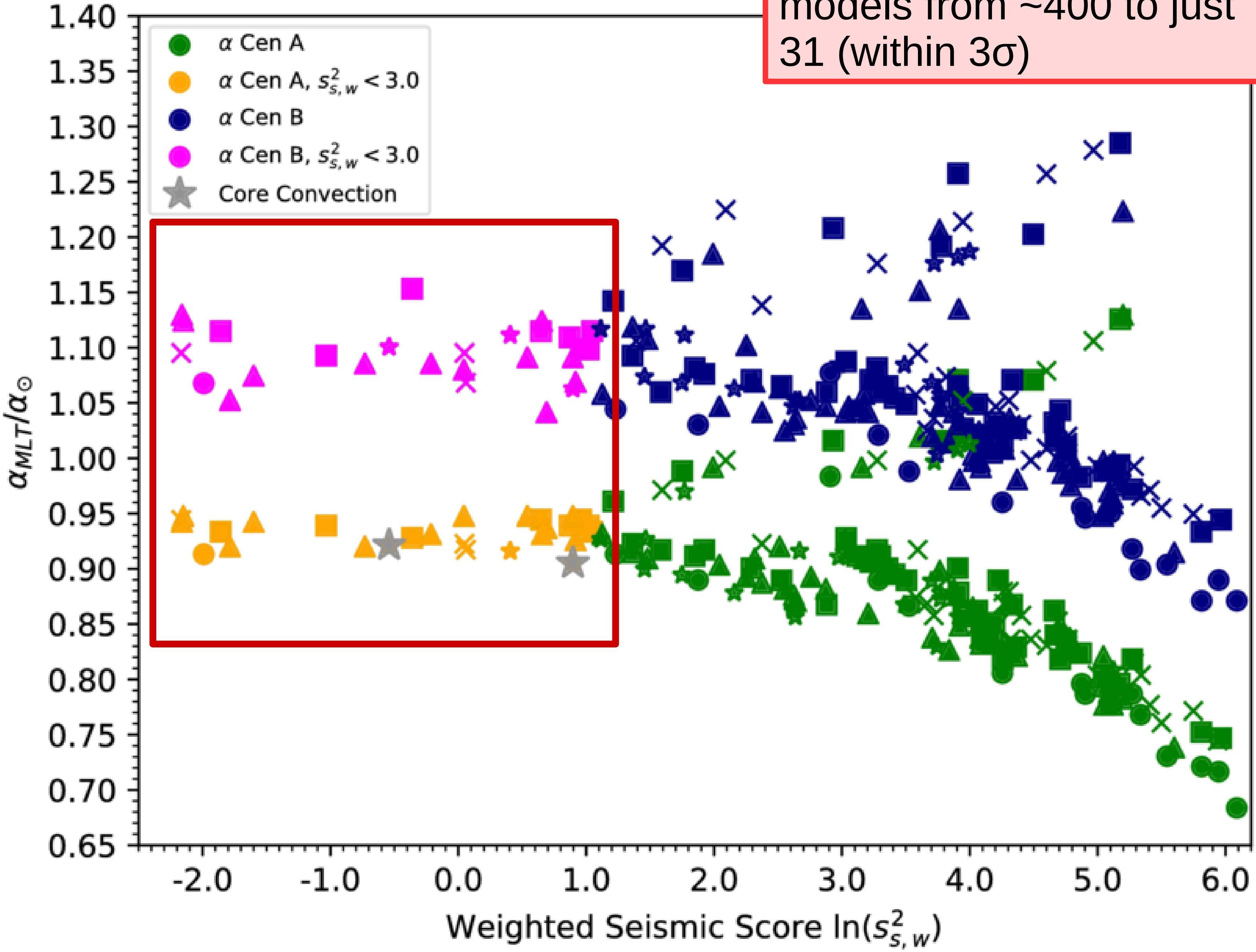
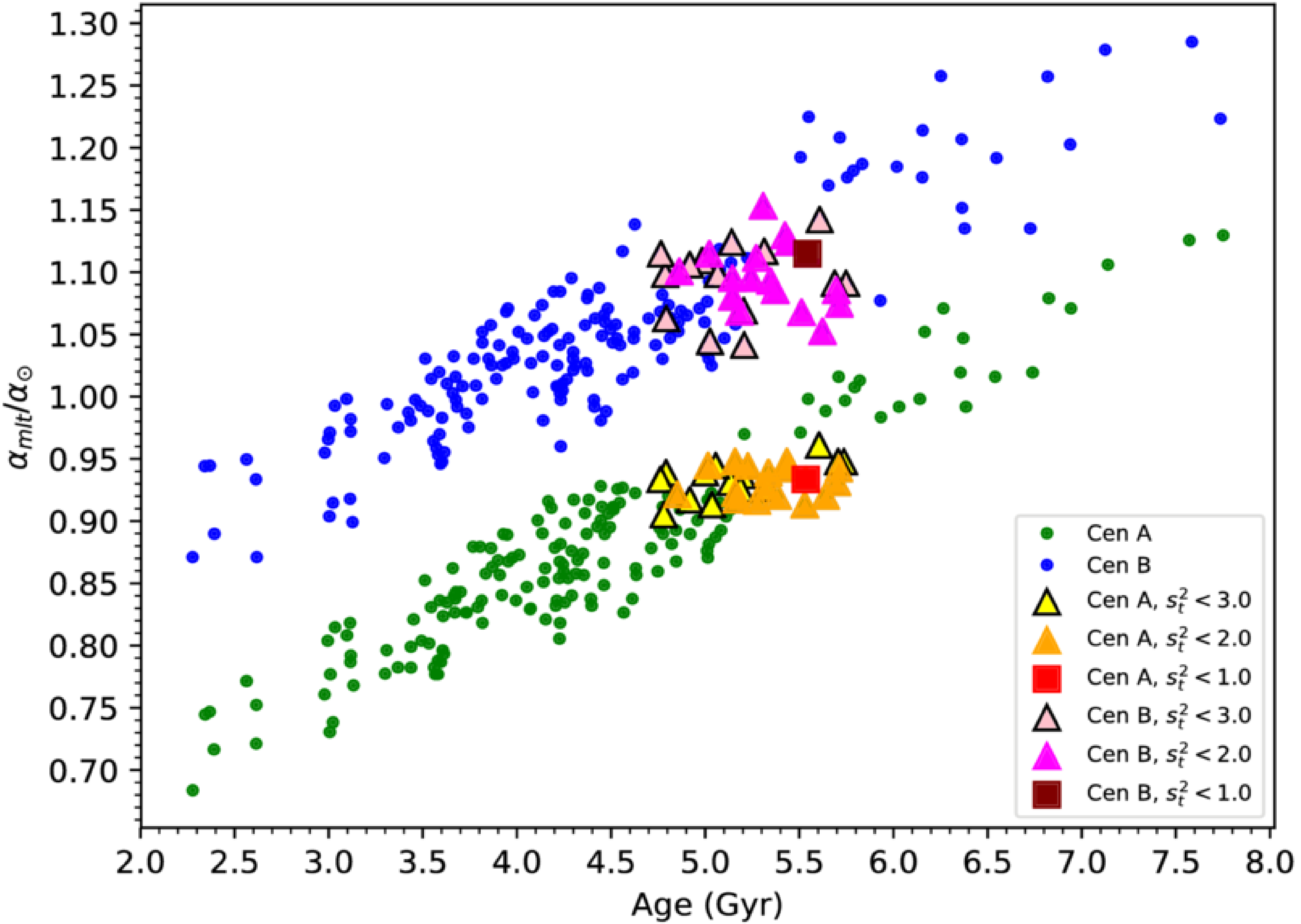


Incorporating seismic constraints:

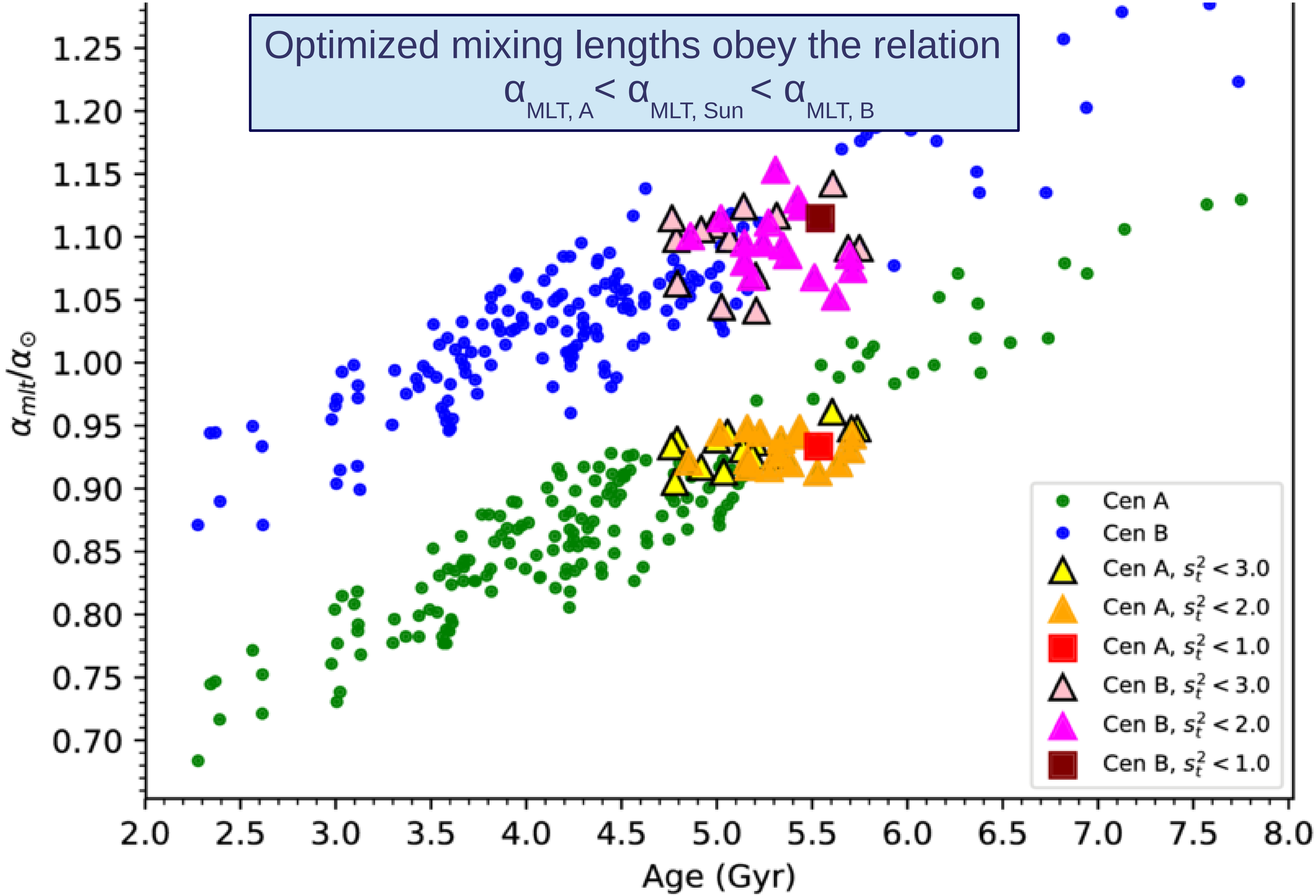
Including seismic criterion reduces number of viable models from ~400 to just 31 (within 3σ)



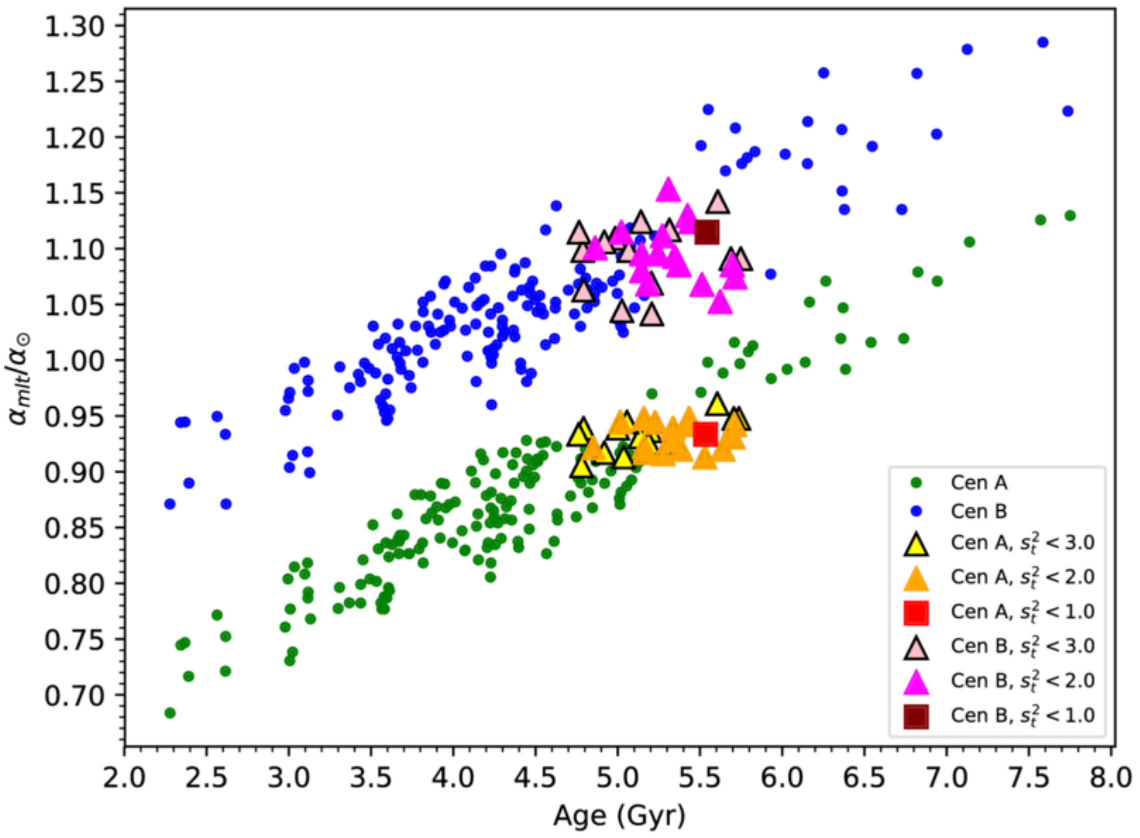
An elegantly converged solution...



From a parameter space including >150,000 models, 31 match all classical and seismic constraints within 3σ



Fundamental Parameters of α Centauri A & B from empirical mixing length calibrations



$$\alpha_{\mathrm{MLT,A}}/\alpha_{\odot} = 0.932 \pm 0.17;$$

$$\alpha_{\mathrm{MLT,B}}/\alpha_{\odot} = 1.095 \pm 0.20;$$

$$t = 5.26 \pm 0.95\,\mathrm{Gyr};$$

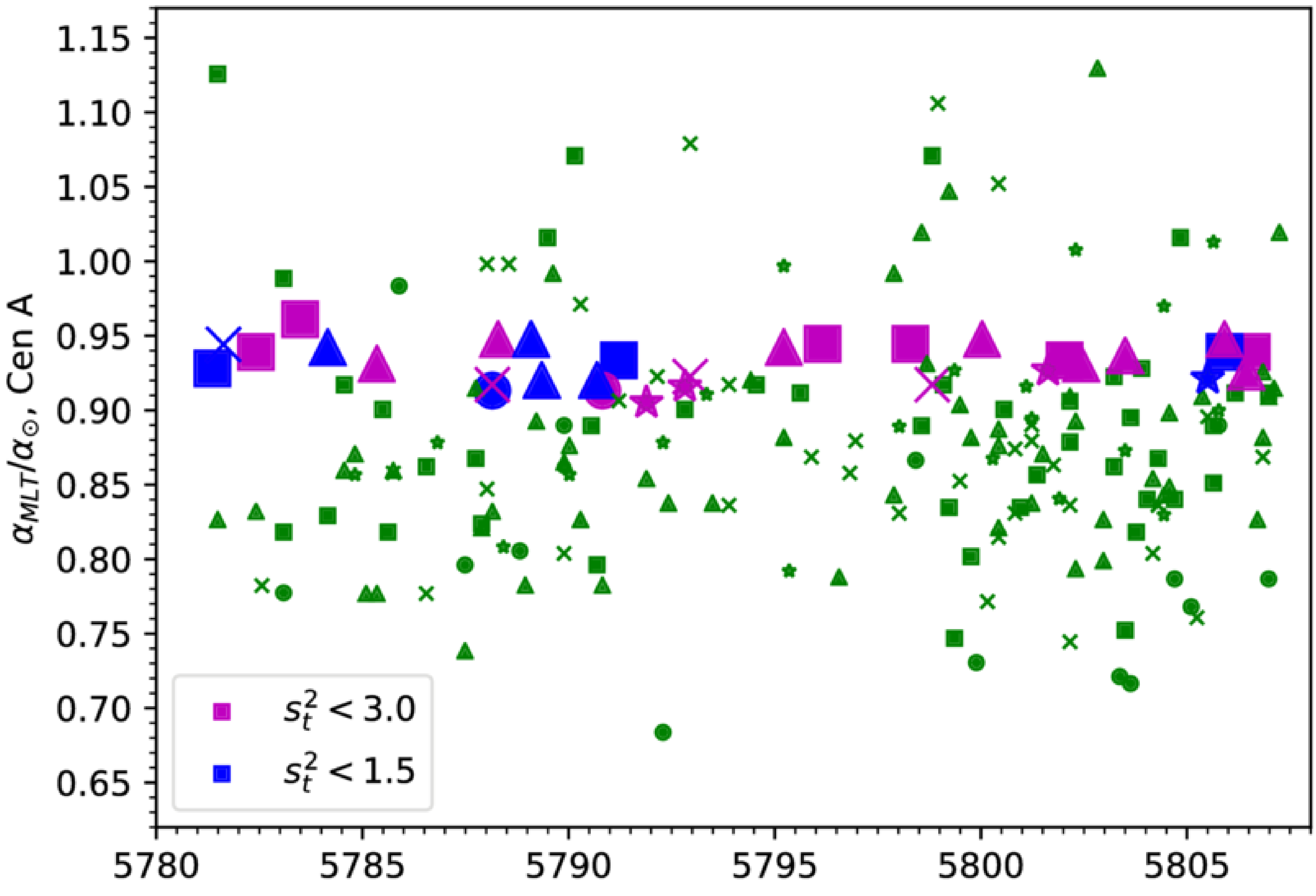
$$\bar{Y}_{\mathrm{in}} = 0.273 \pm 0.035;$$

$$\bar{Z}_{\mathrm{in}} = 0.027 \pm 0.005;$$

$$\Delta Y/\Delta Z = 0.90 \pm 0.12.$$

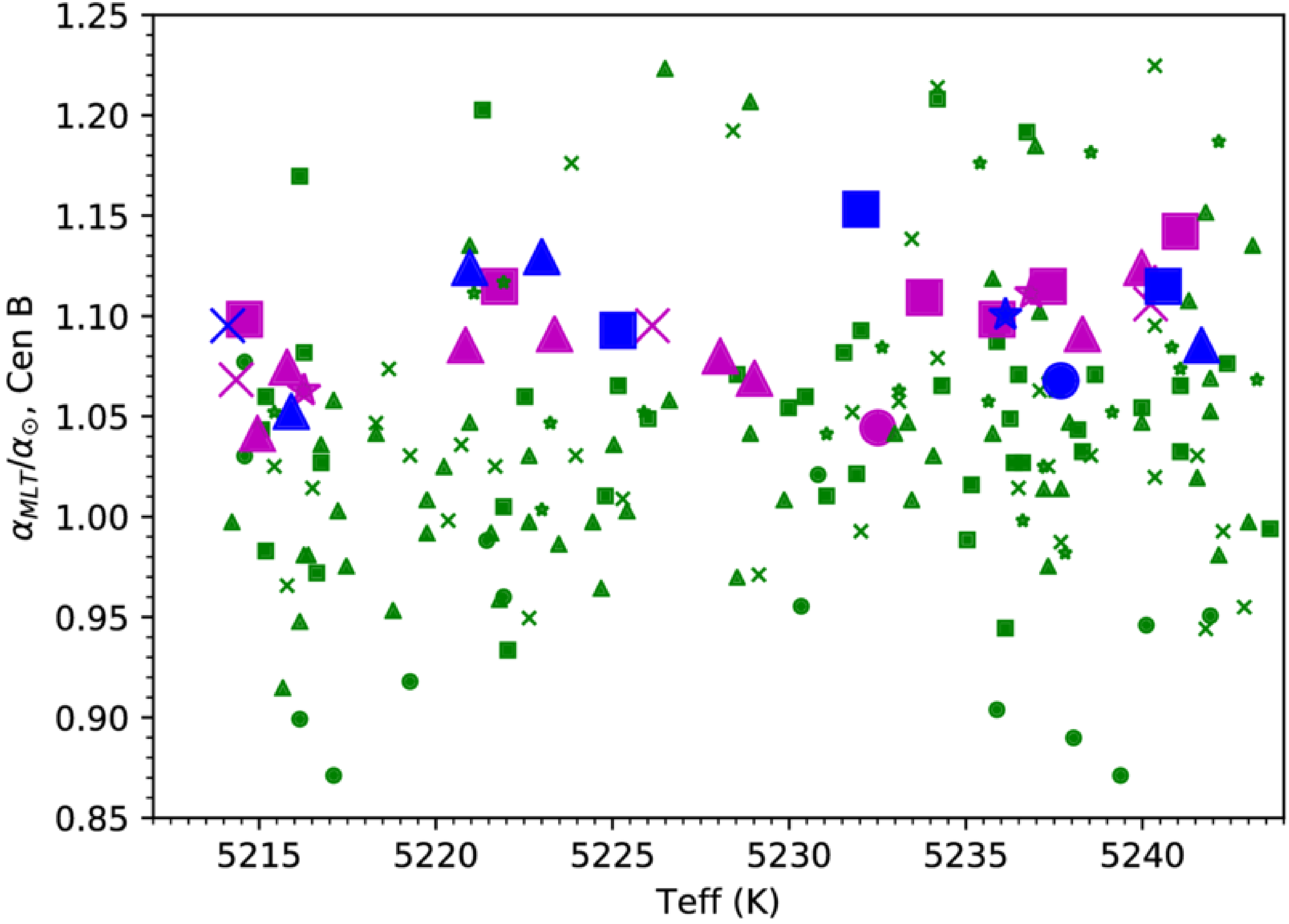
Further insights from fully optimized models:

best-fitting αMLT for **α Cen A** as a function of Teff

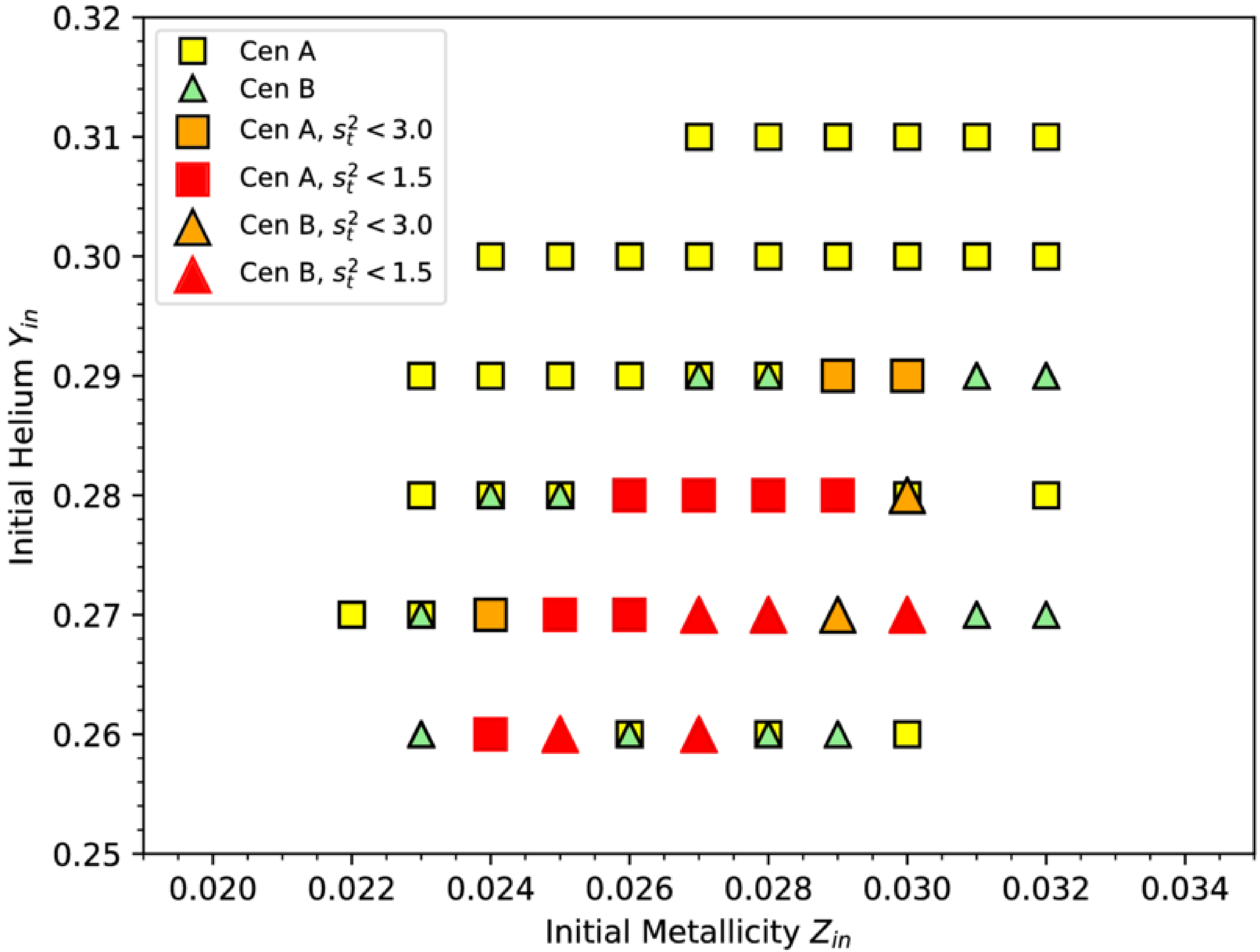


Further insights from fully optimized models:

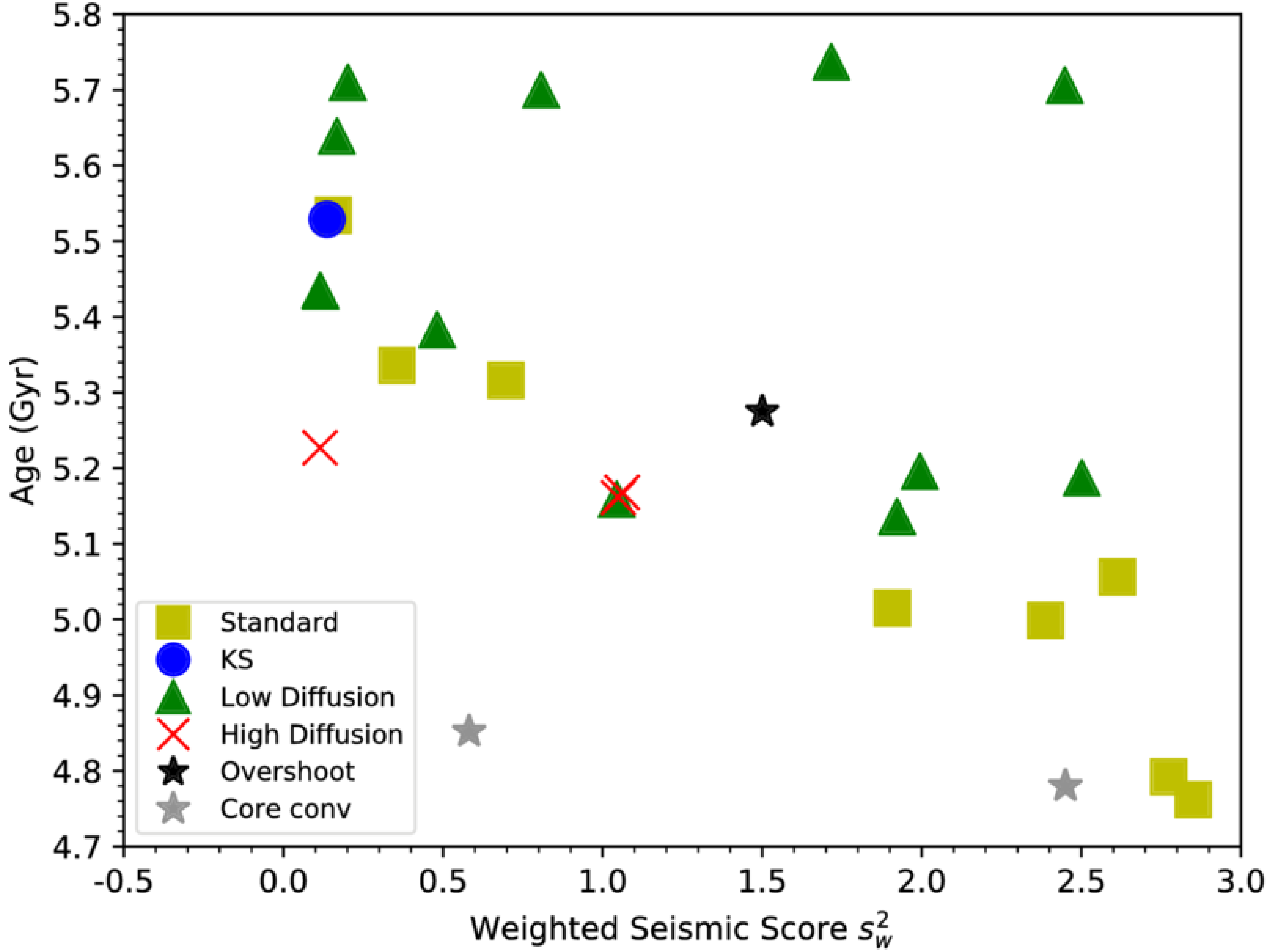
best-fitting αMLT for **α Cen B** as a function of Teff



Further insights from fully optimized models: Composition constraints



We do find two models **compatible with core convection** in alpha Cen A (grey stars)



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+ Asteroseismic

Parameter	
$\alpha_{MLT, A}$	Very tight convergence to 0.93x α_{solar} regardless of choice in modeling physics. Conclusively sub-solar
$\alpha_{MLT, B}$	Converged α_{MLT} is 8-12% **higher** than solar value. Slightly more scatter than estimate for α Cen A
Age	Reduction in the 3 sigma age range by almost an order of magnitude. Age prediction hovers between 4.7 and 5.7 Gyr

General Conclusions:

– seismic constraints severely restrict aMLT, especially for α Cen A

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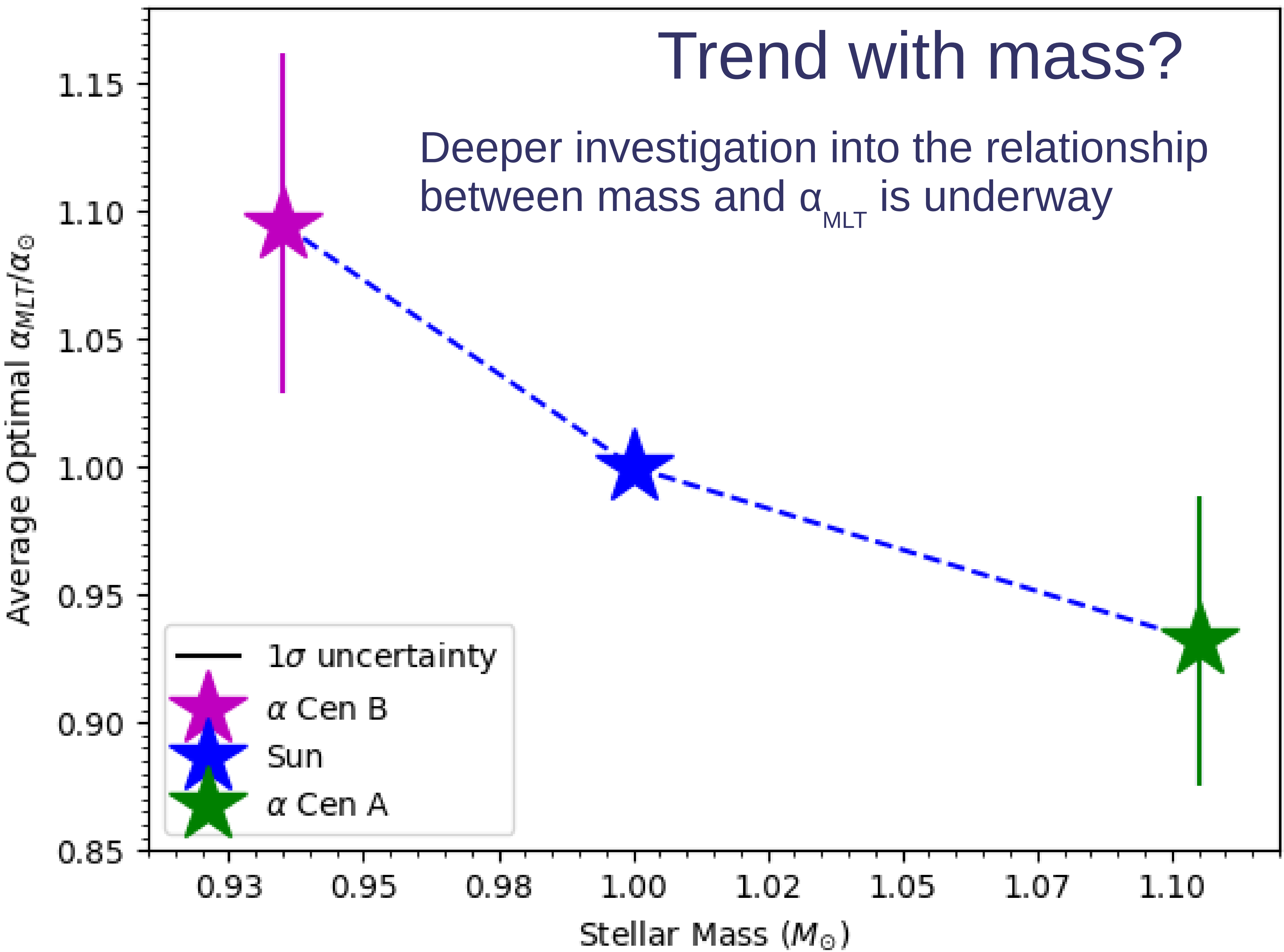
– solar-normalized aMLT converge to well-defined values in both stars!

– globally optimized mixing length values are relatively insensitive to variations in (1D) input physics; main effect is on the age estimate

– under all conditions tested, the hotter and more massive star prefers smaller mixing length values than its cooler, lower-mass counterpart


Trend with mass?
Deeper investigation into the relationship
between mass and α_{MLT} is underway
Average Optimal $\alpha_{MLT}/\alpha_{\odot}$
1.15
1.10
1.05
1.00
0.95
0.90
0.85
0.93
0.95
0.98
1.00
1.02
1.05
1.07
1.10
Stellar Mass ($M_{\odot}$)
1σ uncertainty
α Cen B
Sun
α Cen A

This study lays important groundwork for the future of empirically-informed MLT treatments in stellar modeling!

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→ better parameters for everyone


Merci!